Increased (Platform) Competition Reduces (Seller) Competition

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Abstract

Policymakers have expressed concern that a dominant online platform that acts both as a marketplace and as an active seller might disadvantage its rival sellers and thereby harm consumers. I examine whether platform competition might be promoted to protect consumers. Perhaps surprisingly, I find that increased platform competition can reduce seller competition, and thereby harms consumers.

Key Words: online platforms, platform competition, platform-seller competition.

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1 Introduction

Amazon, as a dominant online platform, not only runs a marketplace for third-party products, but also sells its own private-label products (e.g., Amazon Basics, Amazon Essentials, etc.), in direct competition with third-party sellers. Policymakers have expressed concerns about this practice, in part on the grounds that Amazon might employ proprietary marketplace sales data to determine which private-label products it should develop and market.[1] India introduced legislation in 2019 to prevent Amazon from selling its own private-label products. The United States passed the Ending Platform Monopolies Act in 2021 to limit platform-seller competition between "Big Tech" firms' private-label products and third-party products.

Other authors (e.g., Madsen and Vellodi, 2022; Hagiu, Teh, and Wright, 2022) have examined the welfare implications of policy interventions that limit a dominant platform's ability to employ proprietary sales data or its ability to market private-label products. In contrast, I examine whether platform competition might be promoted to protect consumers. Perhaps surprisingly, I find that increased platform competition often reduces seller competition, and thereby harms consumers.

Increased platform competition can reduce seller competition and harm consumers for the following reason. Potential competition compels a platform to compete for sellers. A platform can do so in my model by effectively committing not to compete against sellers that sell on its platform. I find that when rival platforms have comparable competitive strengths, at least one platform commits not to compete against sellers on its platform and at least one seller faces no competition from the platform. In contrast, a monopoly platform that faces no competition will compete against all sellers on its platform.[2] Therefore, increased platform competition reduces seller competition in the sense that at least one seller faces reduced competition. This reduced seller competition induces higher prices, and thereby harms consumers.

Other studies (e.g., Etro, 2021b; Jeon and Rey, 2021) assume that platforms compete solely by setting the commissions that sellers must pay to the platform on which they sell their products. However, these studies do not consider the case where a platform can harm its rival sellers by selling its own products, in direct competition with its rival sellers, on the platform. In practice, third-party sellers on Amazon list direct competition from Amazon as their most pressing concern.[3] In particular, many third-party sellers with successful products have terminated their online selling due to intense competition from Amazon (Zhu and Liu, 2018). Therefore, in addition to allowing platforms to set commissions, I allow for

[1] The U.S. House Committee on the Judiciary (2020, p.16) asserts that Amazon's dual role as a platform and a seller can motivate the company to engage in anticompetitive conduct. The European Commission (2020) suggests that Amazon's use of non-public seller data to focus its private-label business on the best-selling products is an abuse of a dominant market position.

[2] Amazon has been accused of imitating successful products and selling less expensive competing private-label products (see Mattioli, 2020; Sevilla, 2022; and Cain, 2022).

[3] See Miranda, 2018 and Junglescout, 2020.

the possibility that a platform might attempt to attract third-party sellers by refraining to develop an ability to sell its own products on the platform.

I focus on the setting where platforms choose short-term profit-maximizing commissions after sellers have chosen the platform on which they will operate. I do so in part because, in practice, Amazon does not provide long-term commission contracts and Amazon frequently adjusts its commissions.[4] I also show that my primary finding is not sensitive to the presumed timing. In particular, the same qualitative conclusion arises if platforms can make long-term commitments to both commissions and their selling capabilities before sellers choose the platform on which they will sell their products.

The existing literature suggests that competition between a dominant platform and third-party sellers can either benefit or harm consumers. The competition can benefit consumers because competition for customers leads to lower commissions and retail prices (Etro, 2021a). The competition can harm consumers by inducing higher retail prices because the platform steers consumers towards its products by raising rivals' costs through increased commissions (Anderson and Bedre-Defolie, 2021).[5] The competition can also harm consumers because successful sellers may increase their prices to hide the popularity of their products so as to limit product imitation by platforms (e.g., Jiang, Jerath, and Srinivasan, 2011; Lam and Liu, 2021). I extend this literature to show how increased platform competition affects consumers through its impact on platform-seller competition.

My paper also contributes to the literature on platform competition. Existing papers have focused on network externalities (e.g., Rochet and Tirole, 2003; Armstrong, 2006), seller investment incentives (Belleflamme and Peitz, 2010), advertising (Reisinger, 2012), asymmetric information (Halaburda and Yehezkel, 2013), search diversion (Hagiu and Jullien, 2014), multihoming (Belleflamme and Peitz, 2019), consumer preferences (Bertoletti, 2021), and price coherence (Gerlach and Li, 2021). However, these studies do not consider the case in which platforms sell their own products on their marketplaces, in direct competition with third-party sellers. My paper explains how competition between platforms affects competition between a platform and a third-party seller, and the welfare implications of this competition.

A platform that competes against third-party sellers can be viewed as a vertically integrated supplier that supplies an essential input (i.e., platform access) to its independent downstream rivals. The literature on access pricing discusses the incentives of a vertically integrated supplier to disadvantage its downstream rivals (e.g., Beard, Kaserman, and Mayo, 2001; Mandy and Sappington, 2007). In contrast, I explain how and why competing input suppliers might find it optimal to advantage downstream rivals in order to secure their patronage.[6]

[4] See Hadero, 2022; Hale, 2022; and Kaneshiro, 2020.

[5] See Salop and Scheffman (1983) for a general case in which a predator raises its rivals' costs.

[6] Arya, Mittendorf, and Sappington (2007) find that a vertically integrated wholesale supplier has an incentive to advantage its rival retailer through a reduced wholesale price. In my paper, downstream rivals benefit from reduced competition rather than from reduced input prices.

The analysis proceeds as follows. Section 2 describes the key elements of my model. Sections 3 characterizes outcomes first in the absence of platform entry and then in the presence of platform entry. Section 4 characterizes a benchmark setting with a single monopoly platform. Section 5 characterizes the setting of primary interest, where two platforms compete to attract sellers. Section 6 compares equilibrium outcomes in the settings with and without platform competition. Section 7 discusses extensions. Section 8 provides concluding observations.[7]

2 Model elements

I consider a setting in which two platforms (P1 and P2) compete to attract independent sellers (S1 and S2). Each independent seller sells one product.[8] Each seller can only sell its product on a platform (i.e., the independent seller does not have its own store or website).[9] Each seller can only sell its product on one platform.[10]

I consider a game in which P1 and P2 simultaneously choose to either commit not to act as a seller or make no such commitment. After the commitments are specified, S1 and S2 choose the platform on which they will sell (simultaneously and independently). Next, a platform that made no commitment will make its entry decision (i.e., whether to enter and which market to enter). Then P1 and P2 simultaneously set their per-unit commissions. Finally, each active seller sets its profit-maximizing price for its product.[11] I focus on this timing to reflect the fact that, in practice, platforms often adjust their commissions frequently, and seldom make long-term commitments regarding commissions. Section 7 demonstrates that my primary finding is not sensitive to the presumed timing.[12]

Suppose S1 and S2 sell independent products. Suppose each consumer will purchase one unit of each product. Each seller's type is common knowledge.

If Sj sells on Pk and Pk enters Sj's product market ($k, j \in \{1, 2\}$), Pk imitates Sj's product and Pk and Sj sell differentiated products and compete on prices. Following Singh and Vives (1984), I assume the demands for Pk's product and Sj's product are given by

$$q_{kj}^{P} \;=\; \theta_j\, \alpha_j - \beta_j^{P}\, p_{kj}^{P} + \eta_j\, p_{kj}^{S}, \tag{1}$$

[7] The Appendix outlines the proofs of all formal conclusions. The Technical Appendix provides detailed proofs.

[8] Junglescout (2020) reports that more than a third of sellers have no more than 5 active products listed on Amazon.

[9] Junglescout (2020) reports that 46% of Amazon sellers does not own their own companies.

[10] Amazon launched Amazon Exclusives Program in 2015 to encourage third-party sellers to sell exclusively on Amazon by providing sellers in the program more visibility. I assume each platform provides an exclusive program to attract sellers to sell exclusively on the platform and each seller sells exclusively on one platform to obtain the benefits from the platform's exclusive program.

[11] In the case of multiple sellers, prices are set simultaneously and independently.

[12] In Section 7, I allow platforms to make long-term commitments to both their selling capabilities and the commissions they charge sellers. I show that increased platform competition can continue to harm consumers in this setting.

$$q^S_{kj} \;=\; \alpha_j - \beta^S_j \, p^S_{kj} + \eta_j \, p^P_{kj}. \tag{2}$$

where $\beta^P_j > 0$, $\beta^S_j > 0$, $\eta_j > 0$ are parameters, p^S_{kj} is the price of Sj's product, p^P_{kj} is the price of Pk's product, and α_j represents the popularity of Sj's product.[13] θ_j measures Pk's ability to replicate the popularity of Sj's product. Pk's imitated product can be more or less popular than Sj's product, i.e., $\theta_j \gtreqless 1$. Let $\Omega_j \equiv \frac{[\eta_j]^2}{\beta^P_j \, \beta^S_j} \in (0,1)$ denote the extent to which Sj's product and Pk's product are homogeneous.[14] As Ω_j approaches 0, consumer demands for Sj's product and for Pk's product become nearly independent. As Ω_j approaches 1, consumers view Sj's product and Pk's product as nearly perfect substitutes.

(1) and (2) imply that if Sj sells on Pk and Pk does not enter Sj's product market, then $q^P_{kj} = 0$. Therefore, (1) implies

$$\theta_j \, \alpha_j - \beta^P_j \, p^P_{kj} + \eta_j \, p^S_{kj} \;=\; 0 \;\;\Leftrightarrow\;\; p^P_{kj} \;=\; \frac{\theta_j \, \alpha_j + \eta_j \, p^S_{kj}}{\beta^P_j}. \tag{3}$$

(2) and (3) imply that

$$q^S_{kj} \;=\; \alpha_j - \beta^S_j \, p^S_{kj} + \eta_j \, \frac{\theta_j \, \alpha_j + \eta_j \, p^S_{kj}}{\beta^P_j} \;=\; \alpha_j \left[1 + \frac{\theta_j \, \eta_j}{\beta^P_j} \right] - p^S_{kj} \left[\beta^S_j - \frac{(\eta_j)^2}{\beta^P_j} \right]. \tag{4}$$

(4) implies if Sj sells on Pk, then the demand for Sj's product in the absence of platform entry is:

$$q^S_{kj} \;=\; A_j - b^S_j \, p^S_{kj}, \tag{5}$$

where

$$A_j \;=\; \alpha_j \left[1 + \frac{\theta_j \, \eta_j}{\beta^P_j} \right] \;>\; 0 \;\; \text{and} \;\; b^S_j \;=\; \beta^S_j - \frac{[\eta_j]^2}{\beta^P_j} \;>\; 0. \tag{6}$$

(6) implies that:

$$A_j \;=\; \alpha_j \left[\frac{\beta^P_j + \theta_j \, \eta_j}{\beta^P_j} \right] \;\;\Leftrightarrow\;\; \alpha_j \left[\beta^P_j + \theta_j \, \eta_j \right] \;=\; \beta^P_j \, A_j; \tag{7}$$

$$b^S_j \;=\; \frac{\beta^S_j \, \beta^P_j - [\eta_j]^2}{\beta^P_j} \;\;\Leftrightarrow\;\; \beta^S_j \, \beta^P_j - [\eta_j]^2 \;=\; \beta^P_j \, b^S_j. \tag{8}$$

(6) implies that:

$$\beta^S_j \, \beta^P_j - [\eta_j]^2 \;>\; 0. \tag{9}$$

[13]Pk's product and Sj's product are substitutes, so $\eta_j > 0$.

[14]See Singh and Vives (1984).

(6) also implies that:

$$b_j^S \;=\; \beta_j^S \left[1 - \frac{(\eta_j)^2}{\beta_j^S\, \beta_j^P} \right] \;=\; \beta_j^S \left[1 - \Omega_j \right]. \tag{10}$$

(2) and (5) imply that the impact of platform entry on a seller's sales is ambiguous: (i) platform entry reduces the seller's popularity (i.e., $\alpha_j < A_j$ from (6)) because consumers have more choices; (ii) platform entry increases the sensitivity of the seller's sales to his price (i.e., $\beta_j^P > b_j^S$ from (6)) due to the relatively intense competition; and (iii) platform entry reduces the seller's price due to the relatively intense competition.[15]

Consumers value P1 and P2 differently: a fraction f_1 of consumers are loyal to P1 (i.e., only purchase from P1); a fraction f_2 of consumers are loyal to P2 (i.e., only purchase from P2); and the remainder of the consumers $(1 - f_1 - f_2)$ have no innate preference for P1 and P2. If a seller sells its product at the same price on both platforms, a fraction $f_1 + \frac{1-f_1-f_2}{2}$ of consumers will buy it from P1, and a fraction $f_2 + \frac{1-f_1-f_2}{2}$ of consumers will buy it from P2. If a seller sells its product on P1 and P2 at different prices, a fraction f_k of consumers will buy the product from Pk, and a fraction $1 - f_k$ of consumers will buy the product from Pi, when the product is cheaper on Pi ($i, k \in \{1, 2\}$, $i \neq k$).

If Sj sells on Pk, consumers' demand for Sj's product is ($j, k, i \in \{1, 2\}$, $k \neq i$)

$$Q_{kj}^S \;=\; B_k \left[1 - f_i \right] q_{kj}^S. \tag{11}$$

where $B_k > 1$ is an exogenous "boost" provided by Pk to sales on Pk, and q_{kj}^S is specified in (2) in the presence of platform entry or (5) in the absence of platform entry.

Sj's unit production cost is $c_j^S \geq 0$. If Pk enters Sj's product market, then Pk can imitate Sj's product at cost $c_{kj}^P \geq 0$ ($k, j \in \{1, 2\}$). I assume Pk has sunk fixed cost if Pk does not enter the product market. I assume that each independent seller has zero or sunk fixed cost, whereas Pk must incur a positive fixed cost ($F > 0$) to enter Sj' product market. Pk charges Sj a per-unit commission w_{kj} for each sale.

The ensuing discussion is facilitated by introducing measures of the competitive strengths of the sellers. To begin, let $\Delta_{kj} \equiv \alpha_j - \beta_j^S\, c_j^S + \eta_j\, c_{kj}^P$ denote Sj's "selling strength" and let $\overline{\Delta}_{kj} \equiv \theta_j\, \alpha_j - \beta_j^P\, c_{kj}^P + \eta_j\, c_j^S$ denote Pk's "selling strength" in the presence of platform entry and price competition between Pk and Sj. (1) and (2) imply that Δ_{kj} and $\overline{\Delta}_{kj}$ represent consumers' demand for Sj's product and for Pk's product, respectively, in the presence of platform entry when Pk and Sj price their products at cost under price competition. Thus, Δ_{kj} measures Sj's ability to increase its sales through its product popularity and its relative

[15] A seller's popularity (e.g., A_j in (5)) refers to consumers' demand for the seller's product when the price of the product is zero (e.g., $p_j^S = 0$ in (5)). Therefore, some consumers, who would have purchased the product from the seller in the absence of platform entry, are attracted to the platform's imitated product in the presence of platform entry.

cost advantage, holding prices constant at cost. $\overline{\Delta}_{kj}$ measures the corresponding ability of Pk as an active seller.

Similarly, I define $\widetilde{\Delta}_{kj} \equiv A_j - b_j^S c_j^S$ to be Sj's "selling strength" on Pk in the absence of platform entry, where A_j, and b_j^S are specified in (6). (5) implies that $\widetilde{\Delta}_{kj}$ represents consumers' demand for Sj's product in the absence of platform entry when Sj prices its product at cost. Thus, $\widetilde{\Delta}_{kj}$ measures Sj's ability to increase its sales as a monopolistic seller on Pk holding prices constant at cost.

Furthermore, I define $\Theta_k \equiv B_k [1 - f_i]$ to be Pk's "platform strength" ($j, k, i \in \{1, 2\}$, $k \neq i$). Thus, a platform's (e.g., Pk's) strength increases as it secures more loyal customers (i.e., f_k increases) or its boost (B_k) increases.

(6) implies that

$$\widetilde{\Delta}_{kj} = A_j - b_j^S c_j^S = \alpha_j \left[1 + \frac{\theta_j \eta_j}{\beta_j^P} \right] - \left[\beta_j^S - \frac{(\eta_j)^2}{\beta_j^P} \right] c_j^S$$

$$= \alpha_j - \beta_j^S c_j^S + \frac{\theta_j \eta_j}{\beta_j^P} \alpha_j + \frac{(\eta_j)^2}{\beta_j^P} c_j^S = \Delta_{kj} - \eta_j c_{kj}^P + \frac{\theta_j \eta_j}{\beta_j^P} \alpha_j + \frac{(\eta_j)^2}{\beta_j^P} c_j^S$$

$$= \Delta_{kj} + \eta_j \left[\frac{\theta_j}{\beta_j^P} \alpha_j + \frac{\eta_j}{\beta_j^P} c_j^S - c_{kj}^P \right] = \Delta_{kj} + \eta_j \left[\frac{\theta_j \alpha_j + \eta_j c_j^S - \beta_j^P c_{kj}^P}{\beta_j^P} \right] = \Delta_{kj} + \frac{\eta_j}{\beta_j^P} \overline{\Delta}_{kj}. \tag{12}$$

The ensuing analysis will focus on settings in which $\Delta_{kj} > 0$ and $\overline{\Delta}_{kj} > 0$ (and thus, $\widetilde{\Delta}_{kj} > 0$ from (12)) so that (i) consumers' demand for a seller's product is strictly positive in the absence of platform entry; and (ii) consumers' demands for a seller's product and for a platform's product are strictly positive in the presence of platform entry when the platform and the seller compete on prices.

The key notation is summarized in Table 1.

Table 1 Key Notation

Variables/Symbols	Description
α_j	Sj's popularity in the presence of platform entry
A_j	Sj's popularity in the absence of platform entry
c_j^S	Sj's unit production cost
c_{kj}^P	Pk's unit cost of imitating Sj's product
F	A platform's fixed cost to enter a seller' product market
w_{kj}	Pk's per-unit commission for Sj
p_{kj}^S	Sj's price if Sj sells on Pk
p_{kj}^P	Pk's price when competing against Sj
q_{kj}^S	Initial demand for Sj's product if Sj sells on Pk
q_{kj}^P	Initial demand for Pk's product when Pk enters Sj's market
Q_{kj}^S	Sj's sales if Sj sells on Pk
Q_{kj}^P	Pk's sales if Sj sells on Pk
Θ_k	Pk's platform strength
$\overline{\Delta}_{kj}$	Pk's selling strength as a duopolistic seller
Δ_{kj}	Sj's selling strength on Pk in the presence of platform entry
$\widetilde{\Delta}_{kj}$	Sj's selling strength on Pk in the absence of platform entry

3 Outcomes with and without platform entry

The analysis has two components. Section 3.1 examines the setting where a seller faces no competition from the platform on which it operates.[16] Section 3.2 examines the setting where a seller faces active competition from the platform on which it operates.[17]

To facilitate the statement and proof of key conclusions in the ensuing analysis, it is useful to define the following terms for $k, i, j \in \{1, 2\}$ and $k \neq i$:

$$\Phi_{1j} \equiv 2\,\beta_j^P\,\beta_j^S + \left[\,\eta_j\,\right]^2; \tag{13}$$

$$\Phi_{2j} \equiv 4\,\beta_j^P\,\beta_j^S - \left[\,\eta_j\,\right]^2; \tag{14}$$

$$\Omega_j \equiv \frac{\left[\eta_j\right]^2}{\beta_j^P\,\beta_j^S}; \tag{15}$$

$$M_{kj} \equiv \frac{1}{2\,[\,1-\Omega_j\,]}\left\{\frac{\left[\overline{\Delta}_{kj}\right]^2}{2\,\beta_j^P} + \frac{\Omega_j\,[\,2+\Omega_j\,]\,\left[\,26-\Omega_j+2\,(\Omega_j)^2\,\right]\,[\Delta_{kj}]^2}{2\,\beta_j^S\,[\,8+\Omega_j\,]^2} + \frac{\Omega_j\,\left[\,6+2\,\Omega_j+(\Omega_j)^2\,\right]\,\overline{\Delta}_{kj}\,\Delta_{kj}}{\eta_j\,[\,8+\Omega_j\,]}\right\}; \tag{16}$$

[16]In other words, a seller sells on a platform and the platform does not enter the seller's product market.
[17]In other words, a seller sells on a platform and the platform enters the seller's product market.

$$\phi_{kj} \equiv \left[\left(\frac{8+\Omega_j}{4\sqrt{1-\Omega_j}\,(2+\Omega_j)} \right) \frac{\widetilde{\Delta}_{ij}}{\Delta_{kj}} \right]^2 ; \tag{17}$$

$$\xi_1\left(\frac{\overline{\Delta}_{kj}}{\Delta_{kj}}\right) \equiv \left[\frac{5\,\eta_j\,(2+\Omega_j)}{\beta_j^S\,(8+\Omega_j)} + \frac{\overline{\Delta}_{kj}}{\Delta_{kj}} \right] \left[\frac{\eta_j\,(2+\Omega_j)}{\beta_j^S\,(8+\Omega_j)} + \frac{\overline{\Delta}_{kj}}{\Delta_{kj}} \right] + \frac{4\,\beta_j^P\,[2+\Omega_j]^2}{\beta_j^S\,[8+\Omega_j]^2}; \tag{18}$$

$$\xi_2\left(\frac{\overline{\Delta}_{kj}}{\Delta_{kj}}\right) \equiv \left[\frac{8+\Omega_j}{4\sqrt{1-\Omega_j}\,(2+\Omega_j)} \right]^2 \left[1 + \frac{\eta_j}{\beta_j^P}\,\frac{\overline{\Delta}_{kj}}{\Delta_{kj}} \right]^2 ; \tag{19}$$

$$\overline{a} \equiv \beta_j^S\,[1-\Omega_j]\,,\ \overline{b} \equiv -\frac{4\,\eta_j\,[1-\Omega_j]}{8+\Omega_j}, \text{ and}$$

$$\overline{c} \equiv \frac{\beta_j^P\left[(5\,\Omega_j+4)\,(2+\Omega_j)^2 - (8+\Omega_j)^2\right]}{[8+\Omega_j]^2}. \tag{20}$$

3.1 Outcomes in the absence of platform entry

In this section, I study the setting in which a seller (e.g., Sj) faces no competition from the platform (e.g., Pk) on which it operates.

Lemma 1. *Suppose Sj faces no competition from Pk when Sj sells on Pk ($j,k \in \{1,2\}$). Given w_{kj}, Sj's equilibrium output (i.e., sales) (Q_{kj}^S) is $\frac{\Theta_k\left[\widetilde{\Delta}_{kj} - b_j^S\,w_{kj}\right]}{2}$, and S$j$'s total profit is $\frac{\Theta_k}{b_j^S}\left[q_{kj}^S\right]^2$ where $q_{kj}^S = \frac{Q_{kj}^S}{\Theta_k}$.*

Lemma 1 reports that in the absence of competition from Pk, Sj's profit when it sells its product on Pk increases as: (i) Pk's platform strength (Θ_k) increases; (ii) Sj's output (q_{kj}^S) increases; and (iii) the sensitivity of the demand for Sj's product to its price (b_j^S) declines.[18]

Lemma 1 implies that in the absence of platform entry: (i) Sj will reduce his output (Q_{kj}^S) when he faces a higher commission; and (ii) Sj's sales (Q_{kj}^S) increase as Pk's platform strength (Θ_k) increases and as Sj's selling strength ($\widetilde{\Delta}_{kj}$) increases.

Lemma 2. *Suppose Sj faces no competition from Pk when Sj sells on Pk ($j,k \in \{1,2\}$). Then Pk's profit-maximizing commission for Sj is $w_{kj} = \frac{\widetilde{\Delta}_{kj}}{2\,b_j^S}$.*

Lemma 2 reports that if Pk does not sell on its platform, then Pk's profit-maximizing commission for Sj increases as: (i) Sj's selling strength ($\widetilde{\Delta}_{kj}$) increases; and (ii) the sensitivity of the demand for Sj's product to its price (b_j^S) declines.[19]

[18] (5) implies that b_j^S is the sensitivity of the demand for Sj's product to its price in the absence of platform entry.

[19] (5) implies that b_j^S is the effect of product j's price on its demand in the absence of platform entry.

Lemma 2 implies that in the absence of platform entry, a platform optimally charges the stronger seller a higher commission than it charges the weaker seller. Therefore, the profit-maximizing level of output is less sensitive to the commission for the strong seller.[20]

Lemma 3. *Suppose Sj faces no competition from Pk when Sj sells on Pk ($j,k \in \{1,2\}$). Then Sj's equilibrium output (Q^S_{kj}) is $\frac{\Theta_k \widetilde{\Delta}_{kj}}{4}$, Sj's profit is $\frac{\Theta_k [\widetilde{\Delta}_{kj}]^2}{16\, b^S_j}$, and Pk's profit from the commission it collects from Sj is $\frac{\Theta_k [\widetilde{\Delta}_{kj}]^2}{8\, b^S_j}$.*

Lemma 3 reports that in the absence of platform entry, Sj's equilibrium profit increases as: (i) Pk's platform strength (Θ_k) increases; (ii) Sj's selling strength ($\widetilde{\Delta}_{kj}$) increases; and (iii) the sensitivity of the demand for Sj's product to its price (b^S_j) declines.

3.2 Outcomes in the presence of platform entry

This section considers the setting in which a seller (e.g., Sj) faces active competition from the platform (e.g., Pk) on which it operates ($j,k \in \{1,2\}$). In this setting, Pk first sets its commission w_{kj}, and then Pk and Sj choose their prices (p^P_{kj} and p^S_{kj}), simultaneously and noncooperatively.

Lemma 4. *Suppose Sj competes against Pk when Sj sells on Pk ($j,k \in \{1,2\}$). Given w_{kj}, Sj's equilibrium output (Q^S_j) is $\frac{\Theta_k \left[\frac{\eta_j}{\beta^P_j} \overline{\Delta}_{kj} + 2\,\Delta_{kj} - 2\,\beta^S_j (1-\Omega_j)\, w_{kj} \right]}{4-\Omega_j}$, Pk's equilibrium output (Q^P_{kj}) is $\frac{\Theta_k \left[2\,\overline{\Delta}_{kj} + \frac{\eta_j}{\beta^S_j} \Delta_{kj} - \eta_j (1-\Omega_j)\, w_{kj} \right]}{4-\Omega_j}$, and Sj's total profit is $\frac{\Theta_k}{\beta^S_j} \left[q^S_{kj} \right]^2$ where $q^S_{kj} = \frac{Q^S_{kj}}{\Theta_k}$.*

Lemma 4 reports that when Sj competes against Pk, Sj's profit increases as: (i) Pk's platform strength (Θ_k) increases; (ii) Sj's output (q^S_{kj}) increases; and (iii) the sensitivity of the demand for Sj's product to its price (β^S_j) declines.[21]

Lemma 4 implies that if Pk enters Sj's product market, then Sj will increase his output as the commission he faces (i.e., $\frac{\partial q^S_{kj}}{\partial w_{kj}} < 0$) declines. Furthermore, Pk will sell more of its product in equilibrium when it charges Sj a lower commission (i.e., $\frac{\partial q^P_{kj}}{\partial w_{kj}} < 0$). This is the case because Sj charges a lower price for his product when his costs effectively decline due to a lower commission rate (i.e., $\frac{\partial p^S_{kj}}{\partial w_{kj}} > 0$ from (34)). In response, Pk reduces the price for its product (i.e., $\frac{\partial p^P_{kj}}{\partial w_{kj}} > 0$ and $\frac{\partial p^P_{kj}}{\partial p^S_{kj}} > 0$ from (31)), which causes its equilibrium output to increase.

[20] A seller is considered to be a strong seller when his selling strength is sufficiently pronounced (i.e., the seller sells a highly attractive product or has a low marginal cost). DeGraba (1990) provides related conclusions.

[21] (2) implies that β^S_j is the sensitivity of the demand for Sj's product to its price in the presence of platform entry.

Lemma 5. *Suppose Sj competes against Pk when Sj sells on Pk ($j, k \in \{1, 2\}$). Then Pk's profit-maximizing commission for Sj is*

$$\frac{1}{2[1-\Omega_j]}\left[\frac{\Omega_j\,\overline{\Delta}_{kj}}{\eta_j}+\frac{\left[8+(\Omega_j)^2\right]\Delta_{kj}}{\beta_j^S(8+\Omega_j)}\right].$$

Lemmas 2 and 5 imply that a seller faces a higher commission as its selling strength increases. This is the case because increased seller strength renders the seller's demand for platform access less sensitive to the commission. Consequently, the platform's profit-maximizing commission increases.

Lemma 5 implies that in the presence of competition between a platform and a seller, the commission the seller faces increases as the platform's selling strength increases. This is the case because the platform secures profit both from retail revenue and from commission revenue under platform-seller competition. As the platform's selling strength increases, the platform focuses on securing revenue from its own retail sales. The platform enhances this revenue by increasing the commission it charges to the rival seller, thereby increasing the seller's cost, which induces the seller to raise its retail price. In contrast, as the platform's selling strength declines, the platform focuses on securing commission revenue by encouraging the seller's sales through a reduced commission.

Lemma 6. *Suppose Sj competes against Pk when Sj sells on Pk ($j, k \in \{1, 2\}$). Then Sj's equilibrium output is $\frac{\Theta_k[2+\Omega_j]\Delta_{kj}}{8+\Omega_j}$, Sj's profit is $\frac{\Theta_k}{\beta_j^S}\left[\frac{(2+\Omega_j)\Delta_{kj}}{8+\Omega_j}\right]^2$, Pk's equilibrium output is $\frac{\Theta_k\,\overline{\Delta}_{kj}}{2}+\frac{\eta_j\Theta_k[2+\Omega_j]\Delta_{kj}}{2\beta_j^S[8+\Omega_j]}$, and Pk's profit from the commission it collects from Sj and from entering Sj's market is $\Theta_k\,M_{kj}-F$. Pk sells more than Sj if Pk is a stronger seller than Sj (i.e., $Q_{kj}^P > Q_{kj}^S$ if $\frac{\overline{\Delta}_{kj}}{\Delta_{kj}} > 1$).*

Lemmas 4, 5, and 6 imply that when Pk and Sj compete, Pk optimally charges a commission to offset the effect of Pk's selling strength ($\overline{\Delta}_{kj}$) on Sj's sales and profit, i.e., Sj's equilibrium sales and profit are independent of the selling strength of the platform on which Sj sells.

4 Monopoly Platform (MP)

In this section, I analyze the benchmark setting in which a monopoly platform (P) interacts with two sellers (S1 and S2). The interaction in this setting proceeds as follows. First, P chooses to either commit not to act as a seller or make no such commitment. After the commitment has been made, S1 and S2 choose whether to sell on the platform.[22] If P has made no commitment, then P will make its entry decision (whether to enter and which market to enter). Then, P sets its per-unit commission. Next, each party in each product

[22]Each seller secures zero profit if he chooses not to sell on the platform.

market chooses its own price simultaneously and independently. Without loss of generality, I assume S1 is stronger than S2 ($c_2^S > c_1^S$, and therefore $\Delta_1 > \Delta_2$) in the ensuing discussion.

Condition FS $\Theta_k\, M_{k2} - \frac{\Theta_k \left[\widetilde{\Delta}_{k1}\right]^2}{8\, b_1^S} - \frac{\Theta_k \left[\widetilde{\Delta}_{k2}\right]^2}{8\, b_2^S} \;<\; F \;<\; \min\{\, \Theta_k\, M_{k2} - \frac{\Theta_k \left[\widetilde{\Delta}_{k2}\right]^2}{8\, b_2^S},\, \Theta_k\, M_{k1} - \frac{\Theta_k \left[\widetilde{\Delta}_{k1}\right]^2}{8\, b_1^S} - \frac{\Theta_k \left[\widetilde{\Delta}_{k2}\right]^2}{8\, b_2^S} \}$.

Condition FS ensures that $F \;<\; \Theta_k\, M_{k2} - \frac{\Theta_k \left[\widetilde{\Delta}_{k2}\right]^2}{8\, b_2^S} \;<\; \Theta_k\, M_{k1} - \frac{\Theta_k \left[\widetilde{\Delta}_{k1}\right]^2}{8\, b_1^S}$, i.e., the fixed cost is sufficiently small that the platform will enter both sellers' product market. I assume a platform cannot make discriminating commitments. That is to say, a platform's decision pertains to the markets of both sellers. Thus, if a platform commits not to enter, then it cannot enter S1's market or S2's market. If a platform makes no commitment, then it can enter either seller's market or both markets.

Proposition 1 reports sellers' equilibrium profits and the monopolistic platform's equilibrium profit when Condition FS holds. Proposition 1 implies that in the absence of platform competition, P competes against each seller and secures both commission revenue and retail revenue.

Proposition 1. *Suppose Condition FS holds. In the monopolistic platform setting, both sellers sell on P and P enters both sellers' product markets in equilibrium. Sj's equilibrium profit is* $\frac{\Theta}{\beta_j^S} \left[\frac{(2+\Omega_j)\, \Delta_{Pj}}{8+\Omega_j} \right]^2$*, and P's equilibrium profit is* $\Theta\, M_{P1} + \Theta\, M_{P2} - 2\, F$.

5 Platform Competition (PC)

This section considers the case of primary interest in which two platforms compete to attract two independent sellers.

Platforms can differ in platform strength and selling strength in the product market. Pk's unit cost of imitating Sj's product (i.e., c_{kj}^P) affects both Pk's and Sj's ($k, j \in \{1, 2\}$) selling strengths: as c_{kj}^P increases Pk's cost increases, and therefore, Pk's selling strength declines and Sj's selling strength increases (i.e., $\frac{\partial \overline{\Delta}_{kj}}{\partial c_{kj}^P} < 0$ and $\frac{\partial \Delta_{kj}}{\partial c_{kj}^P} > 0$). Consequently, P1 is a stronger platform than P2 if $\Theta_1 > \Theta_2$. P1 is a stronger seller than P2 (i.e., $\overline{\Delta}_{1j} > \overline{\Delta}_{2j}$) if $c_{1j}^P < c_{2j}^P$.

If both platforms commit not to enter, Sj's ($j \in \{1, 2\}$) selling strength is the same on the two platforms (i.e., $\frac{\partial \widetilde{\Delta}_{kj}}{\partial c_{kj}^P} = 0$). If both platforms provide the same boost to Sj's sales (i.e., $\Theta_1 = \Theta_2$), then Sj is indifferent between selling on P1 and selling on P2. If P1 provides a stronger boost to Sj's sales than does P2 (i.e., $\Theta_1 > \Theta_2$), then Sj will sell on P1. Lemma 7 records this result formally.

Lemma 7. *Suppose Condition FS holds and* $\frac{\Theta_1}{\Theta_2} \geq 1$*. Further suppose both platforms commit not to enter. Then Sj* ($j \in \{1, 2\}$) *is indifferent between selling on P1 and selling on P2 if* $\frac{\Theta_1}{\Theta_2} = 1$*, whereas Sj sells on P1 if* $\frac{\Theta_1}{\Theta_2} > 1$*.*

If neither platform commits not to enter, Condition FS ensures that Sj ($j \in \{1, 2\}$) faces competition from the platform on which Sj sells. In the case where the stronger platform is the weaker seller (i.e., $\Theta_1 > \Theta_2$ and $c^P_{1j} > c^P_{2j}$), Sj sells on P1. This is the case because Sj's selling strength is higher when Sj sells on P1 than when Sj sells on P2 (i.e., $\Delta_{1j} > \Delta_{2j}$) because P1 has a higher cost of imitating Sj's product than does P2 (i.e., $c^P_{1j} > c^P_{2j}$).[23] In addition, P1 provides a stronger boost to Sj's sales than does P2 (i.e., $\Theta_1 > \Theta_2$). In the case where the stronger platform is the stronger seller (i.e., $\Theta_1 > \Theta_2$ and $c^P_{1j} < c^P_{2j}$), Sj faces a trade-off. On the one hand, P1 provides a stronger boost to Sj's sales than does P2 (i.e., $\Theta_1 > \Theta_2$). On the other hand, Sj faces more intense competition from P1 than from P2. This is the case because P1 faces a relatively low cost of imitating Sj's product (i.e., $c^P_{1j} < c^P_{2j}$), and therefore, P1 is a stronger seller than P2 (i.e., $\overline{\Delta}_{1j} > \overline{\Delta}_{2j}$). Consequently, S$j$ will: (i) sell on P1 if the advantage of selling on a strong platform outweighs the disadvantage of competing against a strong seller (i.e., $\frac{\Theta_1}{\Theta_2} > \left[\frac{\Delta_{2j}}{\Delta_{1j}}\right]^2$); and (ii) sell on P2 if the advantage of competing against a weak seller outweighs the disadvantage of selling on a weak platform (i.e., $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{2j}}{\Delta_{1j}}\right]^2$). Lemma 8 records these results formally.

Lemma 8. *Suppose Condition FS holds and* $\frac{\Theta_1}{\Theta_2} \geq 1$*. Further suppose platforms both make no commitment. If* $\frac{c^P_{2j}}{c^P_{1j}} < 1$*, then Sj will sell on P1. If* $\frac{c^P_{2j}}{c^P_{1j}} > 1$*, then Sj will: (i) sell on P1 when* $\frac{\Theta_1}{\Theta_2} > \left[\frac{\Delta_{2j}}{\Delta_{1j}}\right]^2$*; and (ii) sell on P2 when* $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{2j}}{\Delta_{1j}}\right]^2$*.*

If P1 commits not to enter and P2 makes no such commitment, Condition FS ensures that Sj ($j \in \{1, 2\}$) faces competition from P2 if Sj sells on P2. Therefore, Sj will sell on P1. This is the case because Sj's selling strength is higher as a monopolistic seller (i.e., selling on P1) than as a duopolistic seller (i.e., selling on P2). In addition, P1 provides a stronger boost to Sj's sales than does P2. Lemma 9 records this result formally.

Lemma 9. *Suppose Condition FS holds and* $\frac{\Theta_1}{\Theta_2} \geq 1$*. Further suppose P1 commits not to enter and P2 makes no commitment. Then Sj* ($j \in \{1, 2\}$) *will sell on P1.*

If P2 commits not to enter and P1 makes no commitment, Condition FS ensures that Sj ($j \in \{1, 2\}$) faces competition from P1 if Sj sells on P1. Sj faces a trade-off in this case. P1 provides a stronger boost to Sj's sales than does P2. However, Sj's selling strength is higher as a monopolistic seller (i.e., selling on P2) than as a duopolistic seller (i.e., selling on P1). Therefore, Sj will: (i) sell on P1 if the advantage of selling on a strong platform outweighs

[23] Recall that $\frac{\partial \Delta_{kj}}{\partial c^P_{kj}} > 0$.

the disadvantage of competing against a platform (i.e., $\frac{\Theta_1}{\Theta_2} > \phi_j$); and (ii) sell on P2 if the advantage of no platform-seller competition outweighs the disadvantage of selling on a weak platform (i.e., $\frac{\Theta_1}{\Theta_2} < \phi_j$). Lemma 10 records these results formally.

Lemma 10. *Suppose Condition FS holds and $\frac{\Theta_1}{\Theta_2} \geq 1$. Further suppose P1 makes no commitment and P2 commits to no entry. Then Sj ($j \in \{1,2\}$) will: (i) sell on P1 if $\frac{\Theta_1}{\Theta_2} > \phi_j$; and (ii) sell on P2 if $\frac{\Theta_1}{\Theta_2} < \phi_j$.*

Condition FS ensures that Sj competes against Pk if Sj sells on Pk and Pk makes no commitment ($j, k \in \{1,2\}$). Consequently, Sj faces a trade-off when choosing a platform. On the one hand, the stronger platform provides the stronger boost to Sj's sales. On the other hand, the stronger the platform, the more intense platform-seller competition Sj faces because the stronger platform has less incentive to commit not to enter. Because platforms cannot make discriminating commitments, Pk also faces a trade-off. On the one hand, committing not to enter can enhance the likelihood of attracting sellers. On the other hand, if Sj always sells on Pk regardless of Pk's commitment, Pk secures more profit by making no commitment (and subsequently entering Sj's market) than by committing not to enter Sj's market.

Lemmas 7 and 9 imply that if the strong platform (P1) commits not to enter, then both sellers sell on the strong platform. This is the case because each seller benefits from a strong platform boost and no platform-seller competition when the seller sells on P1.

Lemmas 8 and 10 imply that if the strong platform (P1) makes no commitment, then each seller's platform choice depends on: (i) P2's commitment decision; (ii) P1's relative platform strength ($\frac{\Theta_1}{\Theta_2}$); and (iii) P1's relative selling strength ($\frac{\overline{\Delta}_{1j}}{\overline{\Delta}_{2j}}$). Lemmas 8 and 10 also provide three conclusions in the case where the strong platform (P1) makes no commitment. First, if P1's relative platform strength is sufficiently pronounced (i.e., $\frac{\Theta_1}{\Theta_2} > \phi_2$), then each seller sells on the strong platform (P1).[24] This is the case because the advantage of selling on a strong platform outweighs the disadvantage of the platform-seller competition. Second, if P1's relative platform strength is sufficiently limited (i.e., $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2$) and P1's relative selling strength is sufficiently pronounced (i.e., $\frac{\overline{\Delta}_{1j}}{\overline{\Delta}_{2j}} > 1$), then each seller sells on the weak platform (P2).[25] This is the case because the disadvantage of competing against a strong seller outweighs the advantage of selling on a strong platform.[26] Third, if P1's relative platform

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[24] $c_1^S < c_2^S$, $\frac{\partial\left(\frac{\tilde{\Delta}_{2j}}{\Delta_{1j}}\right)}{\partial c_j^S} > 0$, and (17) imply that $\phi_2 > \phi_1$. Therefore, if $\frac{\Theta_1}{\Theta_2} > \phi_2$, then it must be the case that $\frac{\Theta_1}{\Theta_2} > \phi_1$.

[25] Recall that $\frac{\partial \overline{\Delta}_{kj}}{\partial c_{kj}^P} < 0$. Therefore, $\frac{\overline{\Delta}_{1j}}{\overline{\Delta}_{2j}} > 1 \Leftrightarrow \frac{c_{1j}^P}{c_{2j}^P} < 1$. $c_1^S < c_2^S$ and $\frac{\partial\left(\frac{\Delta_{2j}}{\Delta_{1j}}\right)}{\partial c_j^S} > 0$ imply that $\left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2 > \left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2$. Therefore, if $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2$, then it must be the case that $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$.

[26] Because P1's relative selling strength is sufficiently pronounced (i.e., $\frac{\overline{\Delta}_{1j}}{\overline{\Delta}_{2j}} > 1$), P1 is a stronger seller than P2.

strength is relatively pronounced (i.e., $\frac{\Theta_1}{\Theta_2} \in \left(\left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2, \phi_2\right)$), then each seller's platform choice depends on P2's commitment decision and P1's relative selling strength.

Because S2 is a weaker seller than S1 (i.e., $c_1^S < c_2^S$), two conclusions arise. First, the benefit from no platform-seller competition is greater for S2 than for S1 (i.e., $\phi_2 > \phi_1$) when P1 makes no commitment and P2 commits not to enter. Second, the benefit from reduced platform-seller competition is greater for S2 than for S1 (i.e., $\left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2 > \left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2$) when neither platform makes a commitment and P2 is a weaker seller than P1.

I now introduce an assumption that ensures the value of P2's commitment is not trivial in the case where P1 is a stronger seller than P2.[27]

Assumption BC if $c_{1j}^P < c_{2j}^P$, then $\phi_1 > \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$ for $j \in \{1, 2\}$.

Assumption BC pertains to the setting where P1 is a stronger seller than P2 (i.e., $c_{1j}^P < c_{2j}^P$). The assumption states that in this setting, S1's benefit from no platform-seller competition when P1 makes no commitment and P2 commits not to enter exceeds S2's benefit from reduced platform-seller competition when neither platform makes a commitment (i.e., $\phi_1 > \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$).

Proposition 2. *Suppose Condition FS and Assumption BC hold, and $\frac{\Theta_1}{\Theta_2} \geq 1$. Then in equilibrium: (i) if $\frac{\Theta_1}{\Theta_2} > \phi_2$, P1 makes no commitment, and both S1 and S2 sell on P1; (ii) if $\frac{\Theta_1}{\Theta_2} \in (\phi_1, \phi_2)$, P1 makes no commitment whereas P2 commits not to enter, and S1 sells on P1 whereas S2 sells on P2; (iii) if $\frac{\Theta_1}{\Theta_2} \in (1, \phi_1)$, P1 commits not to enter, and both S1 and S2 sell on P1; and (iv) if $\frac{\Theta_1}{\Theta_2} = 1$, both platforms commit not to enter, and each seller is indifferent between selling on P1 and selling on P2.*

[27] In the case where P1 is a stronger seller than P2 (i.e., $c_{1j}^P < c_{2j}^P$), equilibrium results do not vary with the sign of $\phi_1 - \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$ if P1's relative platform strength is sufficiently pronounced (i.e., $\frac{\Theta_1}{\Theta_2} > \phi_2$) or sufficiently limited (i.e., $\frac{\Theta_1}{\Theta_2} < \phi_1$). However, when P1's relative platform strength is relatively pronounced (i.e., $\frac{\Theta_1}{\Theta_2} \in (\phi_1, \phi_2)$), the sign of $\phi_1 - \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$ affects P2's commitment decision in equilibrium. If $\phi_1 > \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$, then S2's benefit from a strong platform boost exceeds the benefit from reduced platform-seller competition (i.e., $\frac{\Theta_1}{\Theta_2} > \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$), but is less than the benefit from no platform-seller competition (i.e., $\frac{\Theta_1}{\Theta_2} < \phi_2$). Therefore, P2 must commit not to enter to ensure that S2 will choose to sell on P2. If $\phi_1 < \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$, P2 also commits not to enter to ensure S2 sells on P2 when $\frac{\Theta_1}{\Theta_2} \in \left(\left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2, \phi_2\right)$. However, P2 can successfully attract S2 without committing not to enter when $\frac{\Theta_1}{\Theta_2} \in \left(\phi_1, \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2\right)$ because S2's benefit from reduced platform-seller competition exceeds the benefit from a strong platform boost (i.e., $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$).

Proposition 2 provides three primary conclusions. First, if P1's relative platform strength is sufficiently pronounced (i.e., if $\frac{\Theta_1}{\Theta_2} > \phi_2$), P1 makes no commitment, and both sellers sell on P1 and compete against P1. This is the case because each seller's benefit from a strong platform boost exceeds the benefit from no platform-seller competition (i.e., $\frac{\Theta_1}{\Theta_2} > \phi_2$ and $\frac{\Theta_1}{\Theta_2} > \phi_1$).[28] Second, as P1's relative platform strength declines (i.e., if $\frac{\Theta_1}{\Theta_2} \in (\phi_1, \phi_2)$), P2 commits not to enter to ensure that S2 will sell on P2. As a weak seller, S2 sells on P2 because S2's benefit from no platform-seller competition exceeds the benefit from a strong platform boost (i.e., $\frac{\Theta_1}{\Theta_2} < \phi_2$). As a strong seller, S1 sells on P1 because S1's benefit from a strong platform boost exceeds the benefit from no platform-seller competition (i.e., $\frac{\Theta_1}{\Theta_2} > \phi_1$). P1 makes no commitment because P1's benefit from imitating the strong seller's (i.e., S1's) popular product exceeds its benefit from attracting an additional seller (i.e., S2). Consequently, S2 faces no competition whereas S1 competes against P1. Third, if P1 and P2 are sufficiently similar (i.e., $\frac{\Theta_1}{\Theta_2} \in (1, \phi_1)$) or symmetric (i.e., $\frac{\Theta_1}{\Theta_2} = 1$) platforms, the relatively intense inter-platform competition compels both platforms to commit not to enter, in order to attract sellers. Consequently, both sellers face no competition.

Figure 1 illustrates these equilibrium outcomes. The figure indicates that as P1's relative platform strength ($\frac{\Theta_1}{\Theta_2}$) declines, platform-seller competition is reduced in each seller's market, and the reduction occurs more rapidly for the weak seller (S2) than for the strong seller (S1). This is the case because as P1's relative platform strength declines, platform competition becomes relatively intense, which compels platforms to commit not to enter in order to attract sellers. When $\frac{\Theta_1}{\Theta_2} > \phi_2$ (pronounced advantage range), each seller's benefit from a strong platform boost exceeds the benefit from no platform-seller competition (i.e., $\frac{\Theta_1}{\Theta_2} > \phi_j$ for $j \in \{1, 2\}$).[29] Therefore, P2's commitment does not affect sellers' platform choices. When $\frac{\Theta_1}{\Theta_2} \in (\phi_1, \phi_2)$ (moderate advantage range), S2's benefit from no platform-seller competition exceeds the benefit from a strong platform boost (i.e., $\frac{\Theta_1}{\Theta_2} < \phi_2$), whereas S1's benefit from a strong platform boost exceeds the benefit it derives from no platform-seller competition (i.e., $\frac{\Theta_1}{\Theta_2} > \phi_1$). Therefore, P2 can successfully attract S2 by committing not to enter. P1 makes no commitment because P1's benefit from imitating the strong seller's (i.e., S1) popular product exceeds P1's benefit from attracting an additional seller (i.e., S2). Consequently, platform-seller competition in S2's market is reduced (in the sense that S2 faces no rival seller in equilibrium) as $\frac{\Theta_1}{\Theta_2}$ declines from the pronounced advantage range to the moderate advantage range. When $\frac{\Theta_1}{\Theta_2} < \phi_1$ (limited advantage range), each seller's benefit from no platform-seller competition exceeds the benefit from a strong platform boost (i.e., $\frac{\Theta_1}{\Theta_2} < \phi_j$ for $j \in \{1, 2\}$).[30] Therefore, P2 can successfully attract both sellers by committing not to enter if P1 makes no commitment. This fact compels P1 to commit not to enter in order to attract sellers. Consequently, platform-seller competition in S1's market is reduced (in the sense that S1 faces no rival seller in equilibrium) as $\frac{\Theta_1}{\Theta_2}$ declines from the moderate advantage

[28] Recall that $\phi_1 < \phi_2$.

[29] Recall that the selling strength gain from no platform-seller competition is higher for the weak seller (S2) than for the strong seller (S1) (i.e., $\phi_2 > \phi_1$). Therefore, if $\frac{\Theta_1}{\Theta_2} > \phi_2$, then it must be the case that $\frac{\Theta_1}{\Theta_2} > \phi_1$.

[30] Because $\phi_2 > \phi_1$, $\frac{\Theta_1}{\Theta_2} < \phi_1$ (i.e., S1's benefit from no platform-seller competition exceeds the benefit from a strong platform boost) implies that $\frac{\Theta_1}{\Theta_2} < \phi_2$ (i.e., S2's benefit from no platform-seller competition exceeds the benefit from a strong platform boost).

range to the limited advantage range.

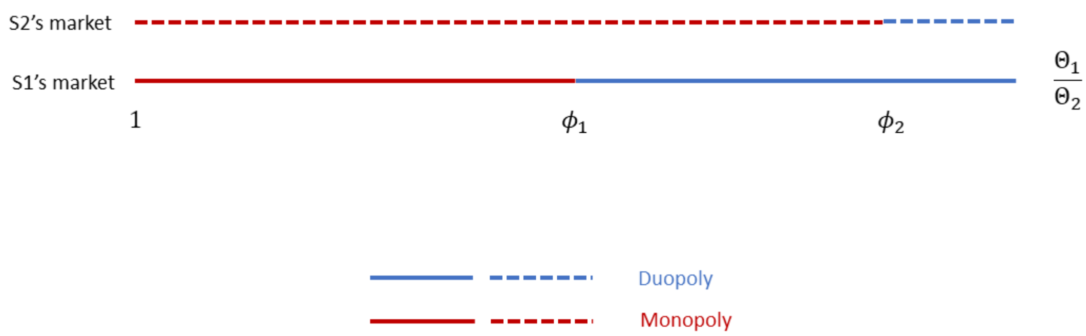

Figure 1: Equilibrium outcomes

6 Comparison of outcomes under MP and PC

In this section, I compare equilibrium outcomes in the benchmark setting with only one platform (MP) and in the setting of primary interest where two platforms compete (PC). In the ensuing discussion, the superscript "M" denotes the MP setting.[31]

When a seller chooses the platform on which it will sell, the seller considers how his sales are affected by platform strengths and the relative intensities of platform-seller competition. Condition FS ensures that Sj ($j \in \{1, 2\}$) competes against P under MP. Increased platform competition might reduce the intensity of platform-seller competition in Sj's market because each platform's commitment decision under PC varies with the competitive strength (which encompasses platform strength and selling strength) of the rival platform.

A sufficiently weak rival platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} < \frac{1}{\phi_2}$) will not affect sellers' platform choices under PC. Therefore, to ensure that the effect of platform competition is not trivial, the ensuing analysis focuses on settings in which the rival platform's ($\widetilde{\text{P}}$'s) relative platform strength is not too limited (i.e., $\frac{\widetilde{\Theta}}{\Theta} > \frac{1}{\phi_2}$).[32] The ensuing analysis focuses on settings in which the rival platform's ($\widetilde{\text{P}}$'s) relative selling strength is weak (i.e., $\frac{\tilde{c}_j^P}{c_j^P} > 1$). The case where $\widetilde{\text{P}}$'s relative selling strength is strong (i.e., $\frac{\tilde{c}_j^P}{c_j^P} < 1$) is analyzed and discussed in the Technical Appendix. Figure 2 depicts $\widetilde{\text{P}}$'s relative platform strength and relative selling strength.

[31] For example, π_j^M and $\Pi_\cdot^M$ denote Sj's and P's equilibrium profits under MP.

[32] Proposition 2 reports that if $\widetilde{\text{P}}$ is a sufficiently weaker platform than P (i.e., $\frac{\widetilde{\Theta}}{\Theta} < \frac{1}{\phi_2}$), then both sellers competes against P under MP and under PC. This is the case because each seller's disadvantage from selling on a sufficiently weak platform outweighs the advantage of no platform-seller competition.

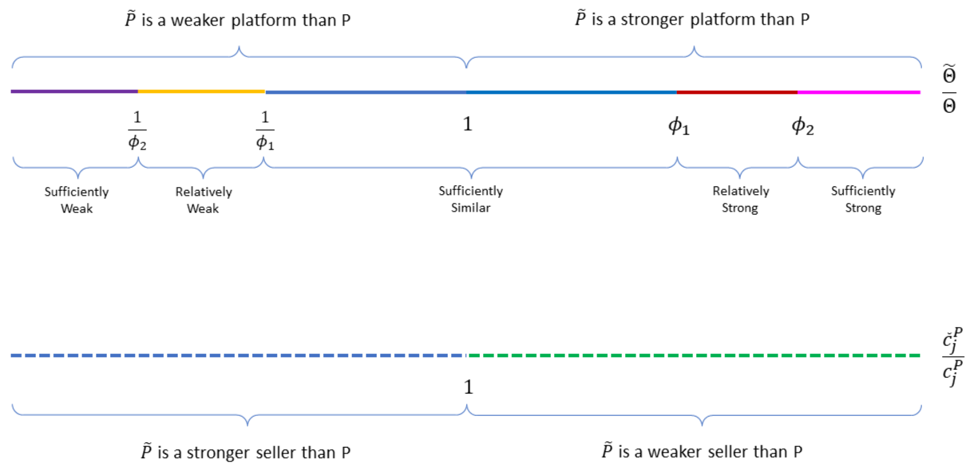

Figure 2: $\widetilde{\mathrm{P}}$'s relative platform strength and relative selling strength

Proposition 2 provides four conclusions. First, if P faces a relatively weak competing platform $\widetilde{\mathrm{P}}$ (i.e., if $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1}\right)$), then $\widetilde{\mathrm{P}}$ commits not to enter to ensure it attracts the weak seller (S2).[33] Given this commitment, S2 sells on $\widetilde{\mathrm{P}}$ because S2's benefit from no platform-seller competition exceeds the benefit from a strong platform boost (i.e., $\phi_2 > \frac{\Theta}{\widetilde{\Theta}}$). P makes no commitment because P's benefit from imitating the strong seller's (i.e., S1) popular product exceeds P's benefit from attracting an additional seller (i.e., S2). S1 sells on P because S1's benefit from a strong platform boost exceeds the benefit from no platform-seller competition (i.e., $\frac{\Theta}{\widetilde{\Theta}} > \phi_1$). Therefore, increased platform competition reduces platform-seller competition in the sense that S2 faces no competition and S1 faces unchanged competition under PC, whereas each seller competes against P under MP.

Second, if P faces a sufficiently similar or symmetric competing platform $\widetilde{\mathrm{P}}$ (i.e., if $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, \phi_1\right)$), then the relatively intense platform competition compels both platforms to commit not to enter in order to attract a seller.[34] Therefore, increased platform competition reduces platform-seller competition in the sense that each seller faces no competition under PC, whereas each seller competes against P under MP.

Third, if P faces a relatively strong competing platform $\widetilde{\mathrm{P}}$ (i.e., if $\frac{\widetilde{\Theta}}{\Theta} \in (\phi_1, \phi_2)$), then P commits not to enter to ensure it attracts the weak seller (S2). Given this commitment, S2 sells on P because S2's benefit from no platform-seller competition exceeds the benefit from a strong platform boost (i.e., $\phi_2 > \frac{\widetilde{\Theta}}{\Theta}$). $\widetilde{\mathrm{P}}$ makes no commitment because $\widetilde{\mathrm{P}}$'s benefit from imitating the strong seller's (i.e., S1) popular product exceeds $\widetilde{\mathrm{P}}$'s benefit from attracting an additional seller (i.e., S2). S1 sells on $\widetilde{\mathrm{P}}$ because S1's benefit from a strong platform

[33] $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1}\right)$ implies that $\frac{\Theta}{\widetilde{\Theta}} \in (\phi_1, \phi_2)$.

[34] $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, \phi_1\right)$ implies that $\frac{\Theta}{\widetilde{\Theta}} < \phi_1$ and $\frac{\widetilde{\Theta}}{\Theta} < \phi_1$.

boost exceeds the benefit from no platform-seller competition (i.e., $\frac{\widetilde{\Theta}}{\Theta} > \phi_1$). Consequently, because $\widetilde{\text{P}}$ is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$), increased platform competition reduces platform-seller competition in the sense that S1 faces reduced competition and S2 faces no competition under PC, whereas each seller competes against P under MP.[35]

Fourth, if P faces a sufficiently strong competing platform $\widetilde{\text{P}}$ (i.e., if $\frac{\widetilde{\Theta}}{\Theta} > \phi_2$), then $\widetilde{\text{P}}$ makes no commitment and both sellers sell on $\widetilde{\text{P}}$. This is the case because each seller's benefit from a strong platform boost exceeds the benefit from no platform-seller competition (i.e., $\frac{\widetilde{\Theta}}{\Theta} > \phi_j$ for $j \in \{1, 2\}$).[36] Therefore, because $\widetilde{\text{P}}$ is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$), increased platform competition reduces platform-seller competition in the sense that each seller faces reduced competition under PC, whereas each seller competes against P under MP.[37]

Proposition 3 states these results formally.

Proposition 3. *Suppose Condition FS and Assumption BC hold. Further suppose P faces a competing platform $\widetilde{P}$ that is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$). Then increased platform competition reduces platform-seller competition in the sense that at least one seller faces no competition or reduced competition and no seller faces increased competition in the presence of platform competition.*

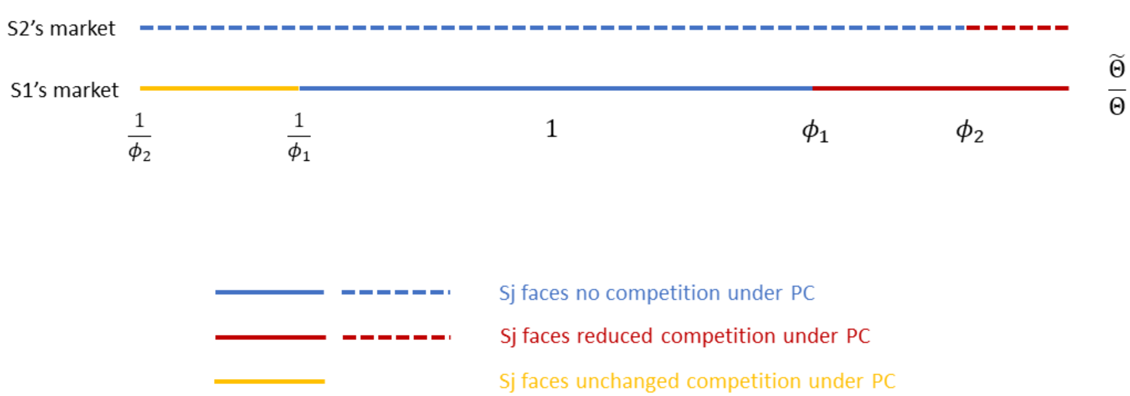

Figure 3: Platform-seller competition under MP and PC

[35] S1 faces reduced competition because S1 competes against a platform that is a weaker seller than P.

[36] Because $\phi_2 > \phi_1$, $\frac{\widetilde{\Theta}}{\Theta} > \phi_2$ ensures that $\frac{\widetilde{\Theta}}{\Theta} > \phi_1$.

[37] Each seller faces reduced competition because each seller competes against a platform that is a weaker seller than P.

Figure 3 compares platform-seller competition under MP and PC. Solid lines represent S1's market and dashed lines represent S2's market. Recall that each seller competes against P under MP. Figure 3 indicates that if $\widetilde{\text{P}}$ is a weaker seller than P, then increased platform competition reduces platform-seller competition in the sense that under PC: (i) one seller faces unchanged competition and the other seller faces no competition (when $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1}\right)$); or (ii) each seller faces no competition (when $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, \phi_1\right)$); or (iii) one seller faces reduced competition and the other seller faces no competition (when $\frac{\widetilde{\Theta}}{\Theta} \in (\phi_1, \phi_2)$); or (iv) each seller faces reduced competition (when $\frac{\widetilde{\Theta}}{\Theta} > \phi_2$).[38]

Proposition 4 reports that increased platform competition either increases or does not change each seller's profit.[39] This is the case because each seller benefits from reduced platform-seller competition or no platform-seller competition under PC.[40]

Proposition 4. *Suppose Condition FS and Assumption BC hold. Further suppose P faces a competing platform $\widetilde{P}$ that is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$). Then $\pi_1 \geq \pi_1^M$ and $\pi_2 > \pi_2^M$, where the first inequality holds strictly unless $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1}\right)$.*

Proposition 5 indicates that if P faces a competing platform that is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$), then increased platform competition reduces consumer surplus (i.e., $CS < CS^M$), unless $\widetilde{\text{P}}$'s relative platform strength is sufficiently pronounced.[41] This is the case because increased platform competition reduces platform-seller competition and reduced platform-seller competition induces higher prices (i.e., $p_1^S \geq p_1^{SM}$ and $p_2^S > p_2^{SM}$). Proposition 6 records this result formally.

Proposition 5. *Suppose Condition FS and Assumption BC hold. Further suppose P faces a competing platform $\widetilde{P}$ that is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$). Then $CS < CS^M$ unless $\widetilde{P}$'s relative platform strength is sufficiently pronounced.*

[38] One seller faces unchanged competition when the seller competes against P both under MP and under PC. One seller faces reduced competition when the seller competes against P under MP whereas the seller competes against a platform that is a weaker seller than P under PC.

[39] Increased platform competition does not change S1's profit when $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1}\right)$ because S1 competes against P both under MP and under PC.

[40] Recall that each seller competes against P under MP. Each seller benefits from reduced platform-seller competition under PC when the seller competes against a platform that is a weaker seller than P. Each seller benefits from no platform-seller competition under PC when the seller sells on a platform that commits not to enter.

[41] If $\widetilde{\text{P}}$'s relative platform strength is sufficiently pronounced (i.e., $\frac{\widetilde{\Theta}}{\Theta} > \max\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\}$), then increased platform competition increases consumer surplus (i.e., $CS > CS^M$). This is the case because the benefit consumers derive from the relatively strong platform boost exceeds the loss consumers suffer from higher prices due to reduced platform-seller competition.

Proposition 6. *Suppose Condition FS and Assumption BC hold. Further suppose P faces a competing platform* $\widetilde{P}$ *that is a weaker seller than P (i.e.,* $\frac{\widetilde{c}_j^P}{c_j^P} > 1$*). Then* $p_1^S \geq p_1^{SM}$ *and* $p_2^S > p_2^{SM}$*, where the first inequality holds strictly unless* $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1}\right)$.

If a seller faces competition from a platform both under MP and under PC, then increased platform competition reduces the commission the seller faces.[42] This is the case because under PC each seller competes against a platform ($\widetilde{\text{P}}$) that is a relatively weak seller. A platform secures profit both from commission revenue and from retail revenue under platform-seller competition. If the platform is a relatively strong seller, then it focuses on securing retail revenue by raising its rival seller's cost and thereby shifting consumers' demand toward its product through a relatively high commission.[43] In contrast, if the platform is a relatively weak seller, then it focuses on securing commission revenue by encouraging the seller's sales through a relatively low commission.

If a seller faces competition from a platform under MP but faces no competition under PC, then increased platform competition increases the commission the seller faces.[44] This is the case because a seller's demand for the access to a platform is less sensitive to the commission as a monopolistic seller than as a duopolistic seller. Therefore, the more inelastic demand for the access to a platform increases the platform's profit-maximizing commission.

If a seller faces unchanged platform-seller competition under MP and under PC, then increased platform competition does not change the prevailing commission.[45] Proposition 7 reports these results formally.

Proposition 7. *Suppose Condition FS and Assumption BC hold. Further suppose P faces a competing platform* $\widetilde{P}$ *that is a weaker seller than P (i.e.,* $\frac{\widetilde{c}_j^P}{c_j^P} > 1$*). Then* $w_j < w_j^M$ *if* $\frac{\widetilde{\Theta}}{\Theta} > \phi_j$ *whereas* $w_j \geq w_j^M$ *if* $\frac{\widetilde{\Theta}}{\Theta} < \phi_j$ *(*$j \in \{1, 2\}$*), where the equality holds when* $j = 1$ *and* $\frac{\widetilde{\Theta}}{\Theta} < \frac{1}{\phi_1}$.

Proposition 8 indicates that if P faces a competing platform that is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$), then increased platform competition reduces total welfare (i.e., $SW < SW^M$) unless $\widetilde{\text{P}}$'s relative platform strength is sufficiently pronounced.[46] This is the case

[42] If $\frac{\widetilde{\Theta}}{\Theta} > \phi_j$, then S$j$ ($j \in \{1, 2\}$) competes against P under MP whereas Sj competes against $\widetilde{\text{P}}$ under PC.

[43] Etro (2021c) provides related discussions.

[44] If $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, \phi_1\right)$, then S1 competes against P under MP whereas S1 faces no competition under PC. If $\frac{\widetilde{\Theta}}{\Theta} < \phi_2$, then S2 competes against P under MP whereas S2 faces no competition under PC.

[45] If $\frac{\widetilde{\Theta}}{\Theta} < \frac{1}{\phi_1}$, then S1 competes against P both under MP and under PC. In this case, $w_1 = w_1^M$ because unchanged platform-seller competition induces no change in the prevailing commission.

[46] If $\widetilde{\text{P}}$'s relative platform strength is sufficiently pronounced, then increased platform competition increases total welfare. This is the case for three primary reasons. First, increased platform competition increases the aggregate profits of platforms if $\widetilde{\text{P}}$'s relative platform strength is sufficiently pronounced. This is

for three primary reasons. First, P suffers from increased platform competition. Second, increased platform competition reduces platform-seller competition and reduced platform-seller competition benefits sellers but hurts consumers due to higher prices. Third, the sum of the loss that platforms suffer from increased platform competition and the loss that consumers suffer from reduced platform-seller competition exceeds the benefit sellers derive from reduced platform-seller competition.

Proposition 8. *Suppose Condition FS and Assumption BC hold. Further suppose P faces a competing platform $\widetilde{\mathrm{P}}$ that is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$). Then $SW < SW^M$ unless $\widetilde{\mathrm{P}}$'s relative platform strength is sufficiently pronounced.*

7 Extension

The foregoing analysis has assumed that platforms cannot make binding long-term commitments to the commissions they charge sellers. I now demonstrate that my primary qualitative conclusion does not rely on this assumption. To do so, I now consider a game in which P1 and P2 simultaneously: (i) choose to either commit not to act as a seller or make no such commitment; and (ii) commit to long-term commissions. After the seller commitments and the commissions are specified, S1 and S2 choose the platform on which they will sell (simultaneously and independently). Next, a platform that made no seller commitment will make its entry decision (i.e., whether to enter and which market to enter). Finally, each active seller sets its profit-maximizing price for its product.

Proposition 9 identifies conditions under which my primary qualitative conclusion - that increased platform competition can harm consumers - persists in this setting. Proposition 9 indicates that when the incumbent platform (P) is a strong seller and platforms are sufficiently similar, then platforms continue to commit not to enter when they can make *ex ante* long-term commitments to both commissions and their selling capabilities. This is the case for two primary reasons. First, the advantage that a seller derives from reduced competition with one platform outweighs the advantage that the seller secures from the lower commission set by the other sufficiently similar platform that competes against sellers. Second, the benefit that consumers derive from platform-seller competition is pronounced when the platform is a strong seller. Consequently, increased platform competition can harm consumers by eliminating the relatively intense competition a strong platform would otherwise supply.

Proposition 9. *Suppose Condition FS holds, P's selling strength relative to its rival seller*

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the case because sellers sell on P under MP and sell on $\widetilde{\mathrm{P}}$ under PC in this case. Therefore, increased platform competition benefits $\widetilde{\mathrm{P}}$ but hurts P and the benefit that $\widetilde{\mathrm{P}}$ derives exceeds the loss that P suffers. This is the case because $\widetilde{\mathrm{P}}$ is a stronger platform and a weaker seller than P and the gain from the relatively pronounced platform strength exceeds the loss from the relatively weak selling strength. Second, Proposition 4 implies that increased platform competition increases the aggregate profits of sellers. Third, Proposition 5 implies that increased platform competition increases consumer surplus if $\widetilde{\mathrm{P}}$'s relative platform strength is sufficiently pronounced.

(Sj) is sufficiently pronounced (i.e., $\frac{\overline{\Delta}_{Pj}}{\Delta_{Pj}} > \frac{-\bar{b}+\sqrt{\left(\bar{b}\right)^2-4\,\bar{a}\,\bar{c}}}{2\,\bar{a}}$*), and* $\widetilde{P}$ *is sufficiently similar to* P *(i.e.,* $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{16[1-\Omega_j][\widetilde{\Delta}_j+\Delta_{Pj}]^2}{[4-\Omega_j]^2[\widetilde{\Delta}_j]^2}, \min\left\{ \frac{\varsigma_{Pj}}{\frac{1}{2\,b_j^S}\left[\frac{\widetilde{\Delta}_j}{2}\right]^2}, \frac{[4-\Omega_j]^2[\widetilde{\Delta}_j]^2}{16[1-\Omega_j][\widetilde{\Delta}_j+\Delta_{\widetilde{P}j}]^2} \right\} \right)$*).*[47] *Further suppose platforms can make ex ante long-term commitments regarding both commissions and their selling capabilities. Then* $CS < CS^M$*.*

Figure 4 illustrates numerical solutions for settings where $\theta_j = 0.9$, $\alpha_j = 30$, $\Theta = 2$, $\beta_j^P = \beta_j^S = 2$, $\eta_j = 1$, $c_1^S = 3$, $c_2^S = 4$.[48] In Figure 4, the yellow areas are those in which $CS < CS^M$ whereas the blue areas are those in which $CS > CS^M$. Panel 1 represents the case where platforms cannot make long-term commitments to commissions, whereas Panel 2 represents the case where platforms can make *ex ante* long-term commitments regarding both commissions and their selling capabilities. The X-axis represents $\frac{c_{P1}^P}{c_1^S}$ (i.e., P's relative selling strength) and the Y-axis represents $\frac{\widetilde{\Theta}}{\Theta}$ (i.e., $\widetilde{\text{P}}$'s relative platform strength).[49] Figure 4 demonstrates that when platforms can make binding commitments to both commissions and their selling capabilities, increased platform competition can reduce consumer surplus even when one platform is almost twice as strong as the other platform and $\frac{c_{P1}^P}{c_1^S}$ is small.

8 Conclusion

Authorities have expressed concerns about a dominant platform competing directly against its third-party sellers. The authorities fear that the dominant platform might disadvantage third-party sellers and thereby harm consumers. I have examined whether increased platform competition might be promoted to protect consumers. I have shown that, rather than protect consumers, increased competition between platforms can reduce competition between platforms and third-party sellers and thereby harm consumers by inducing higher prices.

I have also shown that third-party sellers benefit from reduced seller competition induced by increased platform competition. However, increased platform competition harms consumers more than it benefits sellers, and thereby reduces total welfare.

My findings imply that platform competition does not necessarily benefit consumers. In particular, when a platform competes to attract sellers by declining to develop its own ability to operate as a seller, such competition can harm consumers. This finding does not suggest that platform competition should necessarily be limited or precluded. Rather, the finding suggests that the details of platform competition merit careful study. Some forms of platform

[47] $\bar{a}$, $\bar{b}$, and $\bar{c}$ are as specified in (20).

[48] Parameters are chosen such that $\Omega_j \in (0,1)$, $\Delta_{kj} > 0$, $\overline{\Delta}_{kj} > 0$, and $c_1^S < c_2^S$. $\Theta = 2$ implies that P can double sellers' sales. $\theta_j = 0.9$ implies that platforms' imitated products are slightly less popular than their rival sellers' products. Cui (2022) Part II considers settings where $\theta_j = 1$ (i.e., each platform can perfectly copy its rival seller's popularity) and where $\theta_j = 1.1$ (i.e., platforms' imitated products are slightly more popular than their rival sellers' products).

[49] When $\frac{c_{P1}^P}{c_1^S}$ is small, P's relative selling strength is pronounced.

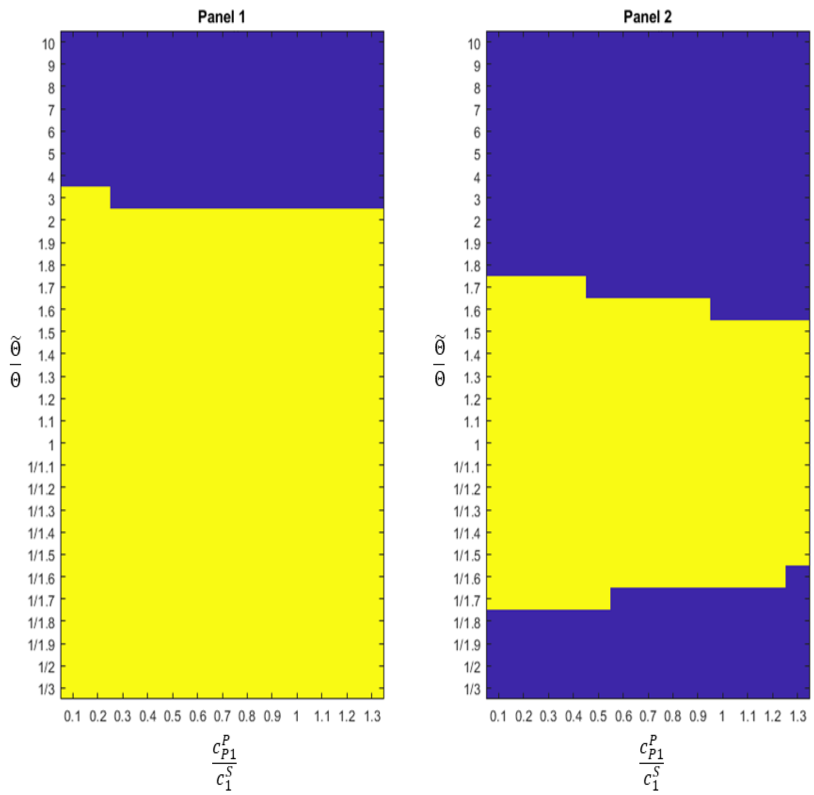

Figure 4: Comparison between CS and CS^M when $\theta_j = 0.9$.

competition (e.g., commission competition) may benefit consumers whereas other forms of competition (e.g., competition on limiting direct selling capabilities) can harm consumers. This is the case because commission competition leads to lower commissions and thereby lower retail prices. In contrast, competition that induces intentional reduction in selling capability can increase retail prices.

I have also shown that a profit-maximizing platform charges a higher commission when the platform declines to develop its selling capability than when the platform is an active seller. This is the case because a seller's demand for access to a platform is more inelastic as a monopolistic seller than as a duopolistic seller.

I have considered settings with two platforms and two sellers for analytic ease. However, my key qualitative conclusions seem likely to persist more generally. For example, increased platform competition can harm consumers in the setting with more than two sellers. This is the case because platform imitation continues to reduce seller profit regardless of the number of sellers. Therefore, platforms continue to compete on commitments to their selling capabilities in order to attract sellers under PC, whereas a monopoly platform continues to imitate successful products regardless of the number of sellers. Furthermore, increased platform competition can harm consumers in the setting with more than two platforms. This is the case because in the presence of increased platform competition, platforms may become more compelled to make commitments to their selling capabilities in order to attract sellers. In this event, increased platform competition will continue to reduce seller competition and thereby harm consumers.

In addition to the extensions discussed in Section 7, future work should consider the possibility that sellers can sell on multiple platforms. I have assumed that each seller only sells its product on one platform. This assumption is consistent with the Amazon Exclusives Program, which encourages exclusive selling by providing more visibility to single-homing sellers. Nonetheless, intense platform competition could induce platforms to accept non-exclusive selling. In this event, the possibility that sellers can sell on multiple platforms might suppress platforms' incentives to commit not to enter. Consequently, increased platform competition might benefit consumers by providing more choices.

Future research should also consider product innovation by third-party sellers.[50] In my analysis, consumers benefit from platform-seller competition through lower prices. However, seller product innovation may be discouraged if platforms effectively imitate successful products. In this event, platform-seller competition might harm consumers. Consequently, increased platform competition that induces platforms to promise not to act as sellers might benefit consumers by stimulating product innovation by third-party sellers.

[50] Platform innovation also merits formal consideration. Platform innovation can affect platform strength. Consequently, increased platform competition that induces platforms to invest in platform innovation might benefit consumers by increasing the boost to sales.

Appendix

This Appendix sketches the proofs of the formal conclusions in the text. Detailed proofs are available in Cui (2022).

Proof of Lemma 1**.** Suppose Sj faces no competition from Pk when Sj sells on Pk ($j, k \in \{1, 2\}$). (11) implies that Sj's profit is given by ($j, k, i \in \{1, 2\}$, $k \neq i$):

$$\pi_j \;=\; \left[\, p^S_{kj} - w_{kj} - c^S_j \,\right] \Theta_k \, q^S_{kj}. \tag{21}$$

(5) and (21) imply Sj chooses p^S_j to maximize π_j:

$$\frac{\partial \pi_j}{\partial p^S_{kj}} \;=\; 0 \quad \Leftrightarrow \quad p^S_{kj}\,(w_{kj}) \;=\; \frac{A_j + b^S_j \, c^S_j + b^S_j \, w_{kj}}{2 \, b^S_j}. \tag{22}$$

(5) and (22) imply that consumers' initial demand for Sj's product is:

$$q^S_{kj} \;=\; \frac{\widetilde{\Delta}_{kj} - b^S_j \, w_{kj}}{2}. \tag{23}$$

(11), (21), (22), and (23) imply that Sj's output is $\frac{\Theta_k \left[\, \widetilde{\Delta}_{kj} - b^S_j \, w_{kj} \,\right]}{2}$ and Sj's profit is $\frac{\Theta_k}{b^S_j} \left[\frac{\widetilde{\Delta}_{kj} - b^S_j \, w_{kj}}{2} \right]^2$. ■

Proof of Lemma 2. Suppose Sj faces no competition from Pk when Sj sells on Pk ($j, k \in \{1, 2\}$). (23) implies that Pk chooses w_{kj} to:

$$\text{Maximize } \; \Pi_k \;=\; w_{kj} \, \Theta_k \, q^S_{kj} + \overline{\Pi}_{kl} \;=\; w_{kj} \, \Theta_k \, \frac{\widetilde{\Delta}_{kj} - b^S_j \, w_{kj}}{2} + \overline{\Pi}_{kl} \tag{24}$$

$$\Rightarrow \quad \frac{\partial \Pi_k}{\partial w_{kj}} \;=\; 0 \quad \Leftrightarrow \quad \frac{\widetilde{\Delta}_{kj} - b^S_j \, w_{kj}}{2} - \frac{b^S_j \, w_{kj}}{2} \;=\; 0 \quad \Leftrightarrow \quad w_{kj} \;=\; \frac{\widetilde{\Delta}_{kj}}{2 \, b^S_j}, \tag{25}$$

where $\overline{\Pi}_{kl}$ is the profit that Pk secures from Sl ($j, k, l \in \{1, 2\}$, $j \neq l$), $\overline{\Pi}_{kl} > 0$ if Sl sells on Pk, and $\overline{\Pi}_{kl} = 0$ if Sl does not on Pk.[51] ■

Proof of Lemma 3. Suppose Sj faces no competition from Pk when Sj sells on Pk ($j, k \in \{1, 2\}$). Lemmas 1 and 2 imply that consumers' initial demand for product j is:

$$q^S_{kj} \;=\; \frac{\widetilde{\Delta}_{kj} - b^S_j \, w_{kj}}{2} \;=\; \frac{\widetilde{\Delta}_{kj} - b^S_j \, \frac{\widetilde{\Delta}_{kj}}{2 \, b^S_j}}{2} \;=\; \frac{\widetilde{\Delta}_{kj} - \frac{\widetilde{\Delta}_{kj}}{2}}{2} \;=\; \frac{\widetilde{\Delta}_{kj}}{4}. \tag{26}$$

[51] $\overline{\Pi}_{kl}$ does not include w_{kj}.

Lemma 1, (11), and (26) imply that Sj's profit is:

$$\pi_j \;=\; \frac{\Theta_k}{b_j^S}\left[\, q_{kj}^S \,\right]^2 \;=\; \frac{\Theta_k}{b_j^S}\left[\frac{\widetilde{\Delta}_{kj}}{4}\right]^2 \;=\; \frac{\Theta_k\left[\,\widetilde{\Delta}_{kj}\,\right]^2}{16\, b_j^S}. \tag{27}$$

Lemma 2 and (26) imply that Pk's profit from charging a commission from Sj is:

$$\Pi_k \;=\; w_{kj}\,\Theta_k\, q_{kj}^S \;=\; \Theta_k\,\frac{\widetilde{\Delta}_{kj}}{2\, b_j^S}\,\frac{\widetilde{\Delta}_{kj}}{4} \;=\; \frac{\Theta_k\left[\,\widetilde{\Delta}_{kj}\,\right]^2}{8\, b_j^S}. \;\blacksquare \tag{28}$$

Proof of Lemma 4. Suppose Sj competes against Pk when Sj sells on Pk ($j, k \in \{1, 2\}$). (1) and (2) imply that Pk's profit is:

$$\Pi_k \;=\; \left[\, p_{kj}^P - c_{kj}^P \,\right]\Theta_k\left[\,\theta_j\,\alpha_j - \beta_j^P\, p_{kj}^P + \eta_j\, p_{kj}^S\,\right] - F + w_{kj}\,\Theta_k\left[\,\alpha_j - \beta_j^S\, p_{kj}^S + \eta_j\, p_{kj}^P\,\right] + \overline{\Pi}_{kl}; \tag{29}$$

Sj's profit is

$$\pi_j \;=\; \left[\, p_{kj}^S - w_{kj} - c_j^S \,\right]\Theta_k\, q_{kj}^S \;=\; \left[\, p_{kj}^S - w_{kj} - c_j^S \,\right]\Theta_k\left[\,\alpha_j - \beta_j^S\, p_{kj}^S + \eta_j\, p_{kj}^P\,\right], \tag{30}$$

where $\overline{\Pi}_{kl}$ is the profit that Pk secures from Sl ($j, k, l \in \{1, 2\}$, $j \neq l$), $\overline{\Pi}_{kl} > 0$ if Sl sells on Pk, and $\overline{\Pi}_{kl} = 0$ if Sl does not on Pk.

(29) implies that Pk chooses its price p_{kj}^P to maximize Π_k:

$$\frac{\partial \Pi_k}{\partial p_{kj}^P} \;=\; 0 \;\;\Leftrightarrow\;\; p_{kj}^P \;=\; \frac{\theta_j\,\alpha_j + \eta_j\, p_{kj}^S + \beta_j^P\, c_{kj}^P + w_{kj}\,\eta_j}{2\,\beta_j^P}. \tag{31}$$

(30) implies that Sj chooses its price p_{kj}^S to maximize π_j:

$$\frac{\partial \pi_j}{\partial p_{kj}^S} \;=\; 0 \;\;\Leftrightarrow\;\; p_{kj}^S \;=\; \frac{\alpha_j + \eta_j\, p_{kj}^P + \beta_j^S\left[\, w_{kj} + c_j^S \,\right]}{2\,\beta_j^S}. \tag{32}$$

where $w_{kj} > 0$ is the commission that Sj faces. (31) and (32) imply that:

$$p_{kj}^P \;=\; \frac{\left[\, 2\,\beta_j^S\,\theta_j + \eta_j \,\right]\alpha_j + 2\,\beta_j^S\,\beta_j^P\, c_{kj}^P + \eta_j\,\beta_j^S\, c_j^S \; + 3\,\beta_j^S\,\eta_j\, w_{kj}}{\Phi_{2kj}}. \tag{33}$$

(32) and (33) imply that:

$$p_{kj}^S \;=\; \frac{\left[\, 2\,\beta_j^P + \eta_j\,\theta_j \,\right]\alpha_j + 2\,\beta_j^P\,\beta_j^S\, c_j^S + \eta_j\,\beta_j^P\, c_{kj}^P}{\Phi_{2kj}} + \frac{w_{kj}}{2}\left[\frac{3\,\left(\eta_j\right)^2}{\Phi_{2kj}} + 1\right]. \tag{34}$$

(2), (33), (14), and (34) imply that consumers' initial demand for product j is:

$$q^S_{kj} \ = \ \frac{\frac{\beta^S_j \, \Omega_j}{\eta_j} \, \overline{\Delta}_{kj} + 2 \, \Delta_{kj} - 2 \, \beta^S_j \, [\, 1 - \Omega_j \,] \; w_{kj}}{4 - \Omega_j}. \tag{35}$$

(11) and (35) imply that Sj's output is $\frac{\Theta_k \left[\frac{\eta_j}{\beta^P_j} \, \overline{\Delta}_{kj} + 2 \, \Delta_{kj} - 2 \, \beta^S_j \, (\, 1 - \Omega_j \,) \, w_{kj} \right]}{4 - \Omega_j}$. (14) and (34) imply that:

$$p^S_{kj} - w_{kj} - c^S_j \ = \ \frac{\frac{\Omega_j}{\eta_j} \, \overline{\Delta}_{kj} + \frac{2}{\beta^S_j} \, \Delta_{kj} - 2 \, [\, 1 - \Omega_j \,] \; w_{kj}}{4 - \Omega_j}. \tag{36}$$

(36) reflects (15). (35) and (36) imply that:

$$p^S_{kj} - w_{kj} - c^S_j \ = \ \frac{q^S_{kj}}{\beta^S_j}. \tag{37}$$

(30), (35), and (37) imply that:

$$\pi_j \ = \ \frac{\Theta_k}{\beta^S_j} \left[\frac{\frac{\beta^S_j \, \Omega_j}{\eta_j} \, \overline{\Delta}_{kj} + 2 \, \Delta_{kj} - 2 \, \beta^S_j \, [\, 1 - \Omega_j \,] \; w_{kj}}{4 - \Omega_j} \right]^2. \tag{38}$$

(1), (33), and (34) imply that:

$$q^P_{kj} \ = \ \theta_j \, \alpha_j - \beta^P_j \, p^P_{kj} + \eta_j \, p^S_{kj} \ = \ \frac{2 \, \overline{\Delta}_{kj} + \frac{\eta_j}{\beta^S_j} \, \Delta_{kj} \ - \eta_j \, w_{kj} \, [\, 1 - \Omega_j \,]}{4 - \Omega_j}. \tag{39}$$

(11) and (39) imply that Pk's output is:

$$Q^P_{kj} \ = \ \Theta_k \, q^P_{kj} \ = \ \frac{\Theta_k \left[2 \, \overline{\Delta}_{kj} + \frac{\eta_j}{\beta^S_j} \, \Delta_{kj} \ - \eta_j \, (\, 1 - \Omega_j \,) \; w_{kj} \right]}{4 - \Omega_j}. \ \blacksquare \tag{40}$$

Proof of Lemma 5. Suppose Sj competes against Pk when Sj sells on Pk ($j, k \in \{1, 2\}$). Pk's profit is

$$\Pi_k \ = \ \left[\, p^P_{kj} - c^P_{kj} \,\right] \Theta_k \, q^P_{kj} - F + w_{kj} \, \Theta_k \, q^S_{kj} + \overline{\Pi}_{kl}, \tag{41}$$

where $\overline{\Pi}_{kl}$ is the profit that Pk secures from Sl ($j, k, l \in \{1, 2\}$, $j \neq l$), $\overline{\Pi}_{kl} > 0$ if Sl sells on Pk, and $\overline{\Pi}_{kl} = 0$ if Sl does not on Pk.[52] Tedious calculations show that:

$$\Pi_k \ = \ \frac{\Theta_k}{\beta^P_j} \left[\, q^P_{kj} \,\right]^2 + w_{kj} \, \Theta_k \left[\frac{\eta_j}{\beta^P_j} \, q^P_{kj} + q^S_{kj} \right] - F + \overline{\Pi}_{kl}. \tag{42}$$

[52] $\overline{\Pi}_{kl}$ does not include w_{kj}.

(42) implies that Pk chooses w_{kj} to

$$\text{Maximize } \Pi_k \;=\; \frac{\Theta_k}{\beta_j^P}\left[\,q_{kj}^P\,\right]^2 + w_{kj}\,\Theta_k\left[\frac{\eta_j}{\beta_j^P}\,q_{kj}^P + q_{kj}^S\right] - F + \overline{\Pi}_{kl} \;\;\Rightarrow\;\; \frac{\partial \Pi_k}{\partial w_{kj}} \;=\; 0, \quad (43)$$

where $\overline{\Pi}_{kl}$ is the profit that Pk secures from Sl ($j,k,l \in \{1,2\}$, $j \neq l$), $\overline{\Pi}_{kl} > 0$ if Sl sells on Pk, $\overline{\Pi}_{kl} = 0$ if Sl does not on Pk, and $\overline{\Pi}_{kl}$ does not include w_{kj}. Tedious calculations show that:

$$w_{kj} \;=\; \frac{1}{2\,[\,1-\Omega_j\,]}\left\{\frac{\Omega_j\,\overline{\Delta}_{kj}}{\eta_j} + \frac{\left[\,8+(\Omega_j)^2\,\right]\,\Delta_{kj}}{\beta_j^S\,[\,8+\Omega_j\,]}\right\}. \;\blacksquare \quad (44)$$

Proof of Lemma 6. Suppose Sj competes against Pk when Sj sells on Pk ($j,k \in \{1,2\}$). Lemmas 4 and 5 imply that Sj's sales are:

$$Q_{kj}^S \;=\; \frac{\Theta_k\left[\frac{\eta_j}{\beta_j^P}\,\overline{\Delta}_{kj} + 2\,\Delta_{kj}\right]}{4-\Omega_j} - \frac{\Theta_k\,2\,\beta_j^S\,[\,1-\Omega_j\,]}{4-\Omega_j}\,w_{kj} \;=\; \frac{\Theta_k\,[\,2+\Omega_j\,]\,\Delta_{kj}}{8+\Omega_j}. \quad (45)$$

(45) implies that

$$q_{kj}^S \;=\; \frac{[\,2+\Omega_j\,]\,\Delta_{kj}}{8+\Omega_j}. \quad (46)$$

Lemma 4 and (46) imply that Sj's profit is

$$\pi_j^S \;=\; \frac{\Theta_k}{\beta_j^S}\left[\,q_{kj}^S\,\right]^2 \;=\; \frac{\Theta_k}{\beta_j^S}\left[\frac{(\,2+\Omega_j\,)\,\Delta_{kj}}{8+\Omega_j}\right]^2. \quad (47)$$

Lemmas 4 and 5 imply that Pk's sales are:

$$Q_{kj}^P \;=\; \frac{\Theta_k\left[\,2\,\overline{\Delta}_{kj} + \frac{\eta_j}{\beta_j^S}\,\Delta_{kj}\right]}{4-\Omega_j} - \frac{\Theta_k\,\eta_j\,[\,1-\Omega_j\,]}{4-\Omega_j}\,w_{kj} \;=\; \frac{\Theta_k\,\overline{\Delta}_{kj}}{2} + \frac{\eta_j\,\Theta_k\,[\,2+\Omega_j\,]\,\Delta_{kj}}{2\beta_j^S\,[\,8+\Omega_j\,]}. \quad (48)$$

(48) implies that:

$$q_{kj}^P \;=\; \frac{\overline{\Delta}_{kj}}{2} + \frac{\eta_j\,[\,2+\Omega_j\,]\,\Delta_{kj}}{2\,\beta_j^S\,[\,8+\Omega_j\,]}. \quad (49)$$

(46) and (48) imply that:

$$Q_{kj}^P \;=\; \frac{\Theta_k\,\overline{\Delta}_{kj}}{2} + \frac{\eta_j}{2\,\beta_j^S}\,Q_{kj}^S. \quad (50)$$

Tedious calculations show that for $\Omega_j \in (0, 1)$,

$$Q^P_{kj} \;>\; Q^S_{kj} \;\text{ if }\; \frac{\overline{\Delta}_{kj}}{\Delta_{kj}} \;>\; 1. \tag{51}$$

(42), (46), (49), and Lemma 5 imply that Pk's profit from the commission it collects from Sj and from entering Sj's product market is:

$$\begin{aligned}\Pi_k \;&=\; \frac{\Theta_k}{\beta^P_j}\left[\, q^P_{kj}\,\right]^2 + w_{kj}\,\Theta_k \left[\, \frac{\eta_j}{\beta^P_j}\, q^P_{kj} + q^S_{kj} \,\right] - F \\ &=\; \frac{\Theta_k}{2\,(\,1-\Omega_j\,)} \left\{ \frac{\left(\overline{\Delta}_{kj}\right)^2}{2\,\beta^P_j} + \frac{(\Delta_{kj})^2\,(\,2+\Omega_j\,)\,\Omega_j}{2\,\beta^S_j\,(\,8+\Omega_j\,)^2}\left[\,26-\Omega_j+2\,(\Omega_j)^2\,\right] \right. \\ &\qquad \left. + \frac{\Omega_j\,\overline{\Delta}_{kj}\,\Delta_{kj}}{\eta_j\,(\,8+\Omega_j\,)}\left[\,6+2\,\Omega_j+(\Omega_j)^2\,\right] \right\} - F. \;\blacksquare\end{aligned}$$

Proof of Proposition 1. Suppose Condition FS holds. Lemma 3 implies that P's profit is $\frac{\Theta\left[\,\widetilde{\Delta}_{Pj}\,\right]^2}{8\,b^S_j}$, if S$j$ ($j \in \{1, 2\}$) sells product j on P and P does not enter Sj's product market. Lemma 6 implies that P's profit is $\Theta\, M_{Pj} - F$, if Sj ($j \in \{1, 2\}$) sells product j on P and P enters Sj's product market. Because Condition FS holds, $\Theta\, M_{Pj} - F > \frac{\Theta\left[\,\widetilde{\Delta}_{Pj}\,\right]^2}{8\,b^S_j}$, i.e., P secures a higher profit by entering Sj's market than "no entry". Therefore, if Sj sells on P, P will enter Sj's market, Lemma 6 implies that P's profit is $\Theta\, M_{Pj} - F$ and Sj's profit is $\frac{\Theta}{\beta^S_j}\left[\frac{(\,2+\Omega_j\,)\,\Delta_{Pj}}{8+\Omega_j}\right]^2$. Therefore, knowing P's entry decisions, Sj ($j \in \{1, 2\}$) will choose to sell on P because he secures a positive profit if he sells on P (i.e., $\frac{\Theta}{\beta^S_j}\left[\frac{(\,2+\Omega_j\,)\,\Delta_{Pj}}{8+\Omega_j}\right]^2 > 0$) while he secures zero profit if he does not sell on P, regardless of the other seller's choice. Therefore, in equilibrium, both S1 and S2 sell on P, and P enters each seller's market. Lemma 6 implies Sj's profit is $\frac{\Theta}{\beta^S_j}\left[\frac{(\,2+\Omega_j\,)\,\Delta_{Pj}}{8+\Omega_j}\right]^2$, and P's profit is $\Theta\, M_{P1} - F + \Theta\, M_{P2} - F$. $\blacksquare$

Proof of Lemma 7. Lemma 3 implies that Sj's profit is $\frac{\Theta_k\left[\widetilde{\Delta}_{kj}\right]^2}{16\,b^S_j}$ if Sj sells on Pk ($j, k \in \{1, 2\}$). (12) implies that

$$\widetilde{\Delta}_{kj} \;=\; \Delta_{kj} + \frac{\eta_j}{\beta^P_j}\,\overline{\Delta}_{kj} \;=\; \alpha_j - \beta^S_j\, c^S_j + \eta_j\, c^P_{kj} + \frac{\eta_j}{\beta^P_j}\left[\,\theta_j\,\alpha_j - \beta^P_j\, c^P_{kj} + \eta_j\, c^S_j\,\right] \tag{52}$$

(52) implies that

$$\frac{\partial \widetilde{\Delta}_{kj}}{\partial c^P_{kj}} \;=\; \eta_j - \frac{\eta_j}{\beta^P_j}\,\beta^P_j \;=\; 0. \tag{53}$$

(53) implies that $\widetilde{\Delta}_{1j} = \widetilde{\Delta}_{2j}$. Consequently, if $\Theta_1 = \Theta_2$, then Sj is indifferent between selling

on P1 and selling on P2; if $\Theta_1 > \Theta_2$, then Sj sells on P1. ■

Proof of Lemma 8. Lemma 6 implies that Sj's profit is $\frac{\Theta_k}{\beta_j^S}\left[\frac{(2+\Omega_j)\Delta_{kj}}{8+\Omega_j}\right]^2$ if Sj sells on Pk $(j, k \in \{1,2\})$. Therefore,

$$\frac{\Theta_1}{\beta_j^S}\left[\frac{(2+\Omega_j)\,\Delta_{1j}}{8+\Omega_j}\right]^2 \gtreqless \frac{\Theta_2}{\beta_j^S}\left[\frac{(2+\Omega_j)\,\Delta_{2j}}{8+\Omega_j}\right]^2 \Leftrightarrow \frac{\Theta_1}{\Theta_2} \gtreqless \left[\frac{\Delta_{2j}}{\Delta_{1j}}\right]^2.$$

Observe that

$$\frac{\Delta_{2j}}{\Delta_{1j}} \gtreqless 1 \Leftrightarrow \frac{c_{2j}^P}{c_{1j}^P} \gtreqless 1.$$

First suppose $\frac{c_{2j}^P}{c_{1j}^P} < 1$. Then S$j$ will sell on P1 because $\frac{\Theta_1}{\Theta_2} \geq 1 > \left[\frac{\Delta_{2j}}{\Delta_{1j}}\right]^2$. Next suppose $\frac{c_{2j}^P}{c_{1j}^P} > 1$. Then S$j$ will: (i) sell on P1 when $\frac{\Theta_1}{\Theta_2} > \left[\frac{\Delta_{2j}}{\Delta_{1j}}\right]^2$; and (ii) sell on P2 when $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{2j}}{\Delta_{1j}}\right]^2$. ■

Proof of Lemma 9. Condition FS ensures that P2 will enter Sj's market if Sj sells on P2 $(j \in \{1,2\})$. Lemma 3 implies that Sj's profit is $\frac{\Theta_1[\widetilde{\Delta}_{1j}]^2}{16\,b_j^S}$ if Sj sells on P1. Lemma 6 implies that Sj's profit is $\frac{\Theta_2}{\beta_j^S}\left[\frac{(2+\Omega_j)\Delta_{2j}}{8+\Omega_j}\right]^2$ if Sj sells on P2. (10) implies that:

$$\frac{\Theta_1\left[\widetilde{\Delta}_{1j}\right]^2}{16\,b_j^S} > \frac{\Theta_2}{\beta_j^S}\left[\frac{(2+\Omega_j)\,\Delta_{2j}}{8+\Omega_j}\right]^2 \Leftrightarrow \frac{\Theta_1}{\Theta_2} > \left[\frac{4\sqrt{1-\Omega_j}\,(2+\Omega_j)}{8+\Omega_j}\frac{\Delta_{2j}}{\widetilde{\Delta}_{1j}}\right]^2. \quad (54)$$

(54) holds because it can be shown that:

$$\frac{\Theta_1}{\Theta_2} \geq 1 > \left[\frac{4\sqrt{1-\Omega_j}\,(2+\Omega_j)}{8+\Omega_j}\frac{\Delta_{2j}}{\widetilde{\Delta}_{1j}}\right]^2. \;■ \quad (55)$$

Proof of Lemma 10. Condition FS ensures that P1 will enter Sj's market if Sj sells on P1 $(j \in \{1,2\})$. Lemma 3 implies that Sj's profit is $\frac{\Theta_2[\widetilde{\Delta}_{2j}]^2}{16\,b_{2j}^S}$ if Sj sells on P2. Lemma 6 implies that Sj's profit is $\frac{\Theta_1}{\beta_j^S}\left[\frac{(2+\Omega_j)\Delta_{1j}}{8+\Omega_j}\right]^2$ if Sj sells on P1. (10) and (17) imply that:

$$\frac{\Theta_1}{\beta_j^S}\left[\frac{(2+\Omega_j)\,\Delta_{1j}}{8+\Omega_j}\right]^2 \gtreqless \frac{\Theta_2\left[\widetilde{\Delta}_{2j}\right]^2}{16\,b_{2j}^S} \Leftrightarrow \frac{\Theta_1}{\Theta_2} \gtreqless \phi_j. \quad (56)$$

It can be shown that:

$$\phi_j > 1. \quad (57)$$

(56) and (57) imply that Sj will: (i) sell on P1 if $\frac{\Theta_1}{\Theta_2} > \phi_j$; and (ii) sell on P2 if $\frac{\Theta_1}{\Theta_2} < \phi_j$. ■

Proof of Proposition 2. Condition FS ensures that each platform enters each seller's market if the platform makes no commitment. Since S1 and S2 sell independent products, S1's choice of platform is independent of S2's choice of platform.

Case I. $c_{1j}^P < c_{2j}^P$.

It can be shown that:

$$\phi_2 \;>\; \phi_1 \;>\; \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2 \;>\; \left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2 \;>\; 1 \;\;\text{if}\;\; c_{1j}^P \;<\; c_{2j}^P. \tag{58}$$

First suppose $\frac{\Theta_1}{\Theta_2} > \phi_2$. Lemmas 7 - 10 and (58) imply that both S1 and S2 sells on P1, regardless of the platforms' commitments. Condition FS ensures P1 enters each seller's market. Therefore, Lemmas 3 and 6 imply that: (i) P1's profit is $\Theta_1 M_{11} - F + \Theta_1 M_{12} - F$ if P1 makes no commitment; and (ii) P1's profit is $\frac{\Theta_1[\widetilde{\Delta}_{11}]^2}{8\,b_1^S} + \frac{\Theta_1[\widetilde{\Delta}_{12}]^2}{8\,b_2^S}$ if P1 commits not to enter. Condition FS ensures that $\Theta_1 M_{1j} - F > \frac{\Theta_1[\widetilde{\Delta}_{1j}]^2}{8\,b_j^S}$, i.e., P1 secures more profit by making no commitment than by committing not to enter. Therefore, in equilibrium, P1 makes no commitment, and both S1 and S2 sell on P1 if $\frac{\Theta_1}{\Theta_2} > \phi_2$.

Next suppose $\frac{\Theta_1}{\Theta_2} \in (\,\phi_1, \phi_2\,)$. Lemmas 7 - 10 and (58) imply that S1 sells on P1, regardless of the platforms' commitments. If P2 makes no commitment, Lemmas 8, 9 and (58) imply that S2 sells on P1. Condition FS and Lemmas 3 and 6 imply that P1 secures more profit by making no commitment than by committing not to enter in this case. If P2 commits not to enter, Lemmas 7 and 10 imply that S2: (i) sells on P1 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Lemmas 3 and 6 imply that P1's profit is: (i) $\frac{\Theta_1[\widetilde{\Delta}_{11}]^2}{8\,b_1^S} + \frac{\Theta_1[\widetilde{\Delta}_{12}]^2}{8\,b_2^S}$ if P1 commits not to enter; and (ii) $\Theta_1 M_{11} - F$ if P1 makes no commitment. Therefore, P1 secures more profit by making no commitment than by committing not to enter in this case because

$$\Theta_1 M_{11} - F \;>\; \frac{\Theta_1\left[\widetilde{\Delta}_{11}\right]^2}{8\,b_1^S} + \frac{\Theta_1\left[\widetilde{\Delta}_{12}\right]^2}{8\,b_2^S} \;\;\Leftrightarrow\;\; F \;<\; \Theta_1 M_{11} - \frac{\Theta_1\left[\widetilde{\Delta}_{11}\right]^2}{8\,b_1^S} - \frac{\Theta_1\left[\widetilde{\Delta}_{12}\right]^2}{8\,b_2^S}.$$

If P1 commits not to enter, Lemmas 7 and 9 imply that S2 sells on P1, regardless of P2's commitment. If P1 makes no commitment, Lemmas 8 and 10 imply that: (i) S2 sells on P1 if P2 makes no commitment; and (ii) S2 sells on P2 if P2 commits not to enter. Therefore, P2 secures more profit by committing not to enter than by making no commitment. Consequently, in equilibrium, P1 makes no commitment whereas P2 commits not to enter, and S1 sells on P1 whereas S2 sells on P2, if $\frac{\Theta_1}{\Theta_2} \in (\,\phi_1, \phi_2\,)$.

Next suppose $\frac{\Theta_1}{\Theta_2} \in \left(\left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2, \phi_1\right)$. If P2 makes no commitment, Lemmas 8, 9 and (58) imply that S1 and S2 both sell on P1, regardless of P1's commitment. Lemmas 3 and 6 imply that P1's profit is: (i) $\frac{\Theta_1 [\widetilde{\Delta}_{11}]^2}{8\, b_1^S} + \frac{\Theta_1 [\widetilde{\Delta}_{12}]^2}{8\, b_2^S}$ if P1 commits not to enter; and (ii) $\Theta_1 M_{11} - F + \Theta_1 M_{12} - F$ if P1 makes no commitment. Condition FS ensures that P1 secures more profit by making no commitment than by committing not to enter in this case. If P2 commits not to enter, Lemmas 7, 10 and (58) imply that Sj ($j \in \{1,2\}$): (i) sells on P1 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Therefore, P1 secures more profit by committing not to enter than by making no commitment in this case. If P1 commits not to enter, Lemmas 7, 9 and (58) imply that both S1 and S2 sell on P1, regardless of P2's commitment. If P1 makes no commitment, Lemmas 8, 10, and (58) imply that Sj ($j \in \{1,2\}$): (i) sells on P1 if P2 makes no commitment; and (ii) sells on P2 if P2 commits not to enter. Therefore, P2 secures more profit by committing not to enter than by making no commitment in this case. Consequently, in equilibrium, both P1 and P2 commit not to enter, and both S1 and S2 sell on P1, if $\frac{\Theta_1}{\Theta_2} \in \left(\left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2, \phi_1 \right)$.

Next suppose $\frac{\Theta_1}{\Theta_2} \in \left(\left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2, \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2 \right)$. If P2 makes no commitment, Lemmas 8, 9, and (58) imply that S1 sells on P1 and S2: (i) sells on P1 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Lemmas 3 and 6 imply that P1's profit is: (i) $\frac{\Theta_1 [\widetilde{\Delta}_{11}]^2}{8\, b_1^S} + \frac{\Theta_1 [\widetilde{\Delta}_{12}]^2}{8\, b_2^S}$ if P1 commits not to enter; and (ii) $\Theta_1 M_{11} - F$ if P1 makes no commitment. Therefore, P1 secures more profit by making no commitment than by committing not to enter in this case because

$$\Theta_1 M_{11} - F \; > \; \frac{\Theta_1 \left[\widetilde{\Delta}_{11}\right]^2}{8\, b_1^S} + \frac{\Theta_1 \left[\widetilde{\Delta}_{12}\right]^2}{8\, b_2^S} \;\; \Leftrightarrow \;\; F \; < \; \Theta_1 M_{11} - \frac{\Theta_1 \left[\widetilde{\Delta}_{11}\right]^2}{8\, b_1^S} - \frac{\Theta_1 \left[\widetilde{\Delta}_{12}\right]^2}{8\, b_2^S}.$$

If P2 commits not to enter, Lemmas 7, 10, and (58) imply that Sj ($j \in \{1,2\}$): (i) sells on P1 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Therefore, P1 secures more profit by committing not to enter than by making no commitment in this case. If P1 commits not to enter, Lemmas 7, 9, and (58) imply that both S1 and S2 sell on P1, regardless of P2's commitment. If P1 makes no commitment, Lemmas 8, 10, and (58) imply that S2 sells on P2 and S1: (i) sells on P1 if P2 makes no commitment; and (ii) sells on P2 if P2 commits not to enter. Lemmas 3 and 6 imply that P2's profit is: (i) $\frac{\Theta_2 [\widetilde{\Delta}_{21}]^2}{8\, b_1^S} + \frac{\Theta_2 [\widetilde{\Delta}_{22}]^2}{8\, b_2^S}$ if P2 commits not to enter; and (ii) $\Theta_2 M_{22} - F$ if P2 makes no commitment. Therefore, P2 secures more profit by committing not to enter than by making no commitment in this case because

$$\Theta_2 M_{22} - F \; < \; \frac{\Theta_2 \left[\widetilde{\Delta}_{21}\right]^2}{8\, b_1^S} + \frac{\Theta_2 \left[\widetilde{\Delta}_{22}\right]^2}{8\, b_2^S} \;\; \Leftrightarrow \;\; F \; > \; \Theta_2 M_{22} - \frac{\Theta_2 \left[\widetilde{\Delta}_{21}\right]^2}{8\, b_1^S} - \frac{\Theta_2 \left[\widetilde{\Delta}_{22}\right]^2}{8\, b_2^S}.$$

Consequently, in equilibrium, both P1 and P2 commit not to enter, and both S1 and S2 sell on P1, if $\frac{\Theta_1}{\Theta_2} \in \left(\left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2, \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2 \right)$.

Next suppose $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2$. If P2 makes no commitment, Lemmas 8, 9, and (58) imply that Sj ($j \in \{1, 2\}$): (i) sells on P1 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Therefore, P1 secures more profit by committing not to enter than by making no commitment in this case. If P2 commits not to enter, Lemmas 7, 10, and (58) imply that Sj ($j \in \{1, 2\}$): (i) sells on P1 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Therefore, P1 secures more profit by committing not to enter than by making no commitment in this case. If P1 commits not to enter, Lemmas 7, 9, and (58) imply that both S1 and S2 sell on P1, regardless of P2's commitment. If P1 makes no commitment, Lemmas 8, 10, and (58) imply that both S1 and S2 sell on P2, regardless of P2's commitment. Lemmas 3 and 6 imply that P2's profit is: (i) $\frac{\Theta_2 [\widetilde{\Delta}_{21}]^2}{8\, b_1^S} + \frac{\Theta_2 [\widetilde{\Delta}_{22}]^2}{8\, b_2^S}$ if P2 commits not to enter; and (ii) $\Theta_2\, M_{21} - F + \Theta_2\, M_{22} - F$ if P2 makes no commitment. Condition FS ensures that P2 secures more profit by making no commitment than by committing not to enter in this case. Consequently, in equilibrium, P1 commits not to enter, and both S1 and S2 sell on P1, if $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2$.

Finally, suppose $\frac{\Theta_1}{\Theta_2} = 1$. Lemma 7 implies that if both platforms commit not to enter, then Sj ($j \in \{1, 2\}$) is indifferent between selling on P1 and selling on P2. Lemma 8 implies that if platforms both make no commitment, then Sj sells on P2. Lemma 9 implies that if P1 commits not to enter and P2 makes no commitment, then Sj sells on P1. Lemma 10 implies that if P1 makes no commitment and P2 commits to no entry, then Sj sells on P2. If P2 makes no commitment, Sj: (i) sells on P1 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Therefore, P1 secures more profit by committing not to enter than by making no commitment in this case. If P2 commits not to enter, Sj: (i) is indifferent between selling on P1 and selling on P2 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Therefore, P1 secures more profit by committing not to enter than by making no commitment in this case. If P1 commits not to enter, Sj: (i) is indifferent between selling on P1 and selling on P2 if P2 commits not to enter; and (ii) sells on P1 if P2 makes no commitment. Therefore, P2 secures more profit by committing not to enter than by making no commitment in this case. Consequently, in equilibrium, both platforms commit not to enter, and each seller is indifferent between selling on P1 and selling on P2, if $\frac{\Theta_1}{\Theta_2} = 1$.

Case II. $c_{1j}^P > c_{2j}^P$.

It can be shown that:

$$\phi_2 \;>\; \phi_1 \;>\; 1 \;>\; \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2 \;>\; \left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2 \;\text{ if }\; c_{1j}^P \;>\; c_{2j}^P. \tag{59}$$

First suppose $\frac{\Theta_1}{\Theta_2} > \phi_2$. Lemmas 7 - 10 and (59) imply that both S1 and S2 sell on P1, regardless of the platforms' commitments. Condition FS ensures P1 enters each seller's market. Therefore, Lemmas 3 and 6 imply that: (i) P1's profit is $\Theta_1\, M_{11} - F + \Theta_1\, M_{12} - F$ if P1 makes no commitment; and (ii) P1's profit is $\frac{\Theta_1 [\widetilde{\Delta}_{11}]^2}{8\, b_1^S} + \frac{\Theta_1 [\widetilde{\Delta}_{12}]^2}{8\, b_2^S}$ if P1 commits not to enter. Condition FS ensures that $\Theta_1 M_{1j} - F > \frac{\Theta_1 [\widetilde{\Delta}_{1j}]^2}{8 b_j^S}$, i.e., P1 secures more profit by making no commitment than by committing to no entry. Therefore, in equilibrium, P1 makes no commitment, and both S1 and S2 sell on P1 if $\frac{\Theta_1}{\Theta_2} > \phi_2$.

Next suppose $\frac{\Theta_1}{\Theta_2} \in (\phi_1, \phi_2)$. Lemmas 7 - 10 and (59) imply that S1 sells on P1, regardless of the platforms' commitments. If P2 makes no commitment, Lemmas 8, 9 imply that S2 sells on P1. Condition FS and Lemmas 3 and 6 imply that P1 secures more profit by making no commitment than by committing not to enter in this case. If P2 commits not to enter, Lemmas 7 and 10 imply that S2: (i) sells on P1 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Lemmas 3 and 6 imply that P1's profit is: (i) $\frac{\Theta_1 [\widetilde{\Delta}_{11}]^2}{8 b_1^S} + \frac{\Theta_1 [\widetilde{\Delta}_{12}]^2}{8 b_2^S}$ if P1 commits not to enter; and (ii) $\Theta_1 M_{11} - F$ if P1 makes no commitment. Therefore, Therefore, P1 secures more profit by making no commitment than by committing not to enter in this case because

$$\Theta_1 M_{11} - F \ > \ \frac{\Theta_1 \left[\widetilde{\Delta}_{11} \right]^2}{8 b_1^S} + \frac{\Theta_1 \left[\widetilde{\Delta}_{12} \right]^2}{8 b_2^S} \ \Leftrightarrow \ F \ < \ \Theta_1 M_{11} - \frac{\Theta_1 \left[\widetilde{\Delta}_{11} \right]^2}{8 b_1^S} - \frac{\Theta_1 \left[\widetilde{\Delta}_{12} \right]^2}{8 b_2^S}.$$

If P1 commits not to enter, Lemmas 7 and 9 imply that S2 sells on P1, regardless of P2's commitment. If P1 makes no commitment, Lemmas 8 and 10 imply that: (i) S2 sells on P1 if P2 makes no commitment; and (ii) S2 sells P2 if P2 commits not to enter. Therefore, P2 secures more profit by committing not to enter than by making no commitment. Consequently, in equilibrium, P1 makes no commitment while P2 commits not to enter, and S1 sells on P1 while S2 sells on P2, if $\frac{\Theta_1}{\Theta_2} \in (\phi_1, \phi_2)$.

Next suppose $\frac{\Theta_1}{\Theta_2} < \phi_1$. If P2 makes no commitment, Lemmas 8 and 9 imply that both S1 and S2 sell on P1, regardless of P1's commitment. Condition FS and Lemmas 3 and 6 imply that P1 secures more profit by making no commitment than by committing not to enter in this case. If P2 commits not to enter, Lemmas 7 and 10 imply that both S1 and S2 sell: (i) on P1 if P1 commits not to enter; and (ii) on P2 if P1 makes no commitment. Therefore, P1 secures more profit by committing not to enter than by making no commitment in this case. If P1 makes no commitment, Lemmas 8 and 10 imply that both S1 and S2 sell: (i) on P1 if P2 makes no commitment; and (ii) on P2 if P2 commits not to enter. Therefore, P2 secures more profit by committing not to enter than by making no commitment. If P1 commits not to enter, Lemmas 7 and 9 imply that both S1 and S2 sell on P1, regardless of P2's commitment. Therefore, in equilibrium, both P1 and P2 commit not to enter and both S1 and S2 sell on P1 if $\frac{\Theta_1}{\Theta_2} < \phi_1$.

Finally, suppose $\frac{\Theta_1}{\Theta_2} = 1$. Lemma 7 implies that if both platforms commit not to enter, then Sj ($j \in \{1, 2\}$) is indifferent between selling on P1 and selling on P2. Lemma 8 implies that if platforms both make no commitment, then Sj sells on P1. Lemma 9 implies that if P1 commits not to enter and P2 makes no commitment, then Sj sells on P1. Lemma 10 implies that if P1 makes no commitment and P2 commits to no entry, then Sj sells on P2. If P2 makes no commitment, Sj sells on P1, regardless of P1's commitment. If P2 commits not to enter, Sj: (i) is indifferent between selling on P1 and selling on P2 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Therefore, P1 secures more profit by committing not to enter than by making no commitment in this case. If P1 commits not to enter, Sj: (i) is indifferent between selling on P1 and selling on P2 if P2 commits not to enter; and (ii) sells on P1 if P2 makes no commitment. Therefore, P2 secures more profit by committing not to enter than by making no commitment in this case. If P1 makes no commitment, Sj: sells on P1 if P2 makes no commitment; and (ii) sells on P2 if P2 commits to no entry. Therefore, P2 secures more profit by committing not to enter than by making no commitment in this case. Consequently, in equilibrium, both platforms commit not to enter, and each seller is indifferent between selling on P1 and selling on P2, if $\frac{\Theta_1}{\Theta_2} = 1$. ■

Proof of Proposition 3. Proposition 1 implies that Sj ($j \in \{1, 2\}$) competes against P under MP. First suppose P faces a competing platform $\widetilde{\text{P}}$ that is a stronger platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} > 1$) under PC, where $\widetilde{\Theta}$ denotes $\widetilde{\text{P}}$'s platform strength.

Case I. $\frac{\widetilde{\Theta}}{\Theta} > \phi_2$.

Proposition 2 implies that each seller competes against $\widetilde{\text{P}}$ under PC. Because $\widetilde{\text{P}}$ is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$), increased platform competition reduces platform-seller competition in the sense that each seller faces reduced competition under PC, whereas each seller competes against P under MP.[53]

Case II. $\frac{\widetilde{\Theta}}{\Theta} \in (\, \phi_1, \phi_2 \,)$.

Proposition 2 implies that S1 competes against $\widetilde{\text{P}}$ whereas S2 faces no competition under PC. Because $\widetilde{\text{P}}$ is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$), increased platform competition reduces platform-seller competition in the sense that S1 faces reduced competition and S2 faces no competition under PC, whereas each seller competes against P under MP.

Case III. $\frac{\widetilde{\Theta}}{\Theta} \in (\, 1, \phi_1 \,)$.

Proposition 2 implies that each seller faces no competition under PC. Therefore, increased platform competition reduces platform-seller competition in the sense that each seller faces no competition under PC, whereas each seller competes against P under MP.

Next suppose P faces a symmetric competing platform $\widetilde{\text{P}}$ (i.e., $\frac{\widetilde{\Theta}}{\Theta} = 1$) under PC. Proposition 2 implies that each seller faces no competition under PC. Therefore, increased platform competition reduces platform-seller competition in the sense that each seller faces no competition under PC, whereas each seller competes against P under MP. Finally, suppose P faces a competing platform $\widetilde{\text{P}}$ that is a weaker platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} < 1$) under PC.

Case I. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, 1 \right)$.

[53] $\widetilde{c}_j^P$ denotes $\widetilde{\text{P}}$'s cost of imitating Sj's product, and c_j^P denotes P's cost of imitating Sj's product.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\, 1, \phi_1 \,)$. Proposition 2 implies that each seller faces no competition under PC. Therefore, increased platform competition reduces platform-seller competition in the sense that each seller faces no competition under PC, whereas each seller competes against P under MP.

Case II. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1} \right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\, \phi_1, \phi_2 \,)$. Proposition 2 implies that S1 competes against P whereas S2 faces no competition under PC. Therefore, increased platform competition reduces platform-seller competition in the sense that S2 faces no competition and S1 faces unchanged competition under PC, whereas each seller competes against P under MP. ■

Proof of Proposition 4. Proposition 1 indicates that Sj's ($j \in \{1, 2\}$) equilibrium profit under MP is

$$\pi_j^M \;=\; \frac{\Theta}{\beta_j^S} \left[\frac{(\, 2 + \Omega_j \,)\, \Delta_{Pj}}{8 + \Omega_j} \right]^2 . \tag{60}$$

First suppose P faces a competing platform $\widetilde{\text{P}}$ that is a stronger platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} > 1$) under PC.

Case I. $\frac{\widetilde{\Theta}}{\Theta} > \phi_2$.

Proposition 2 implies that each seller competes against $\widetilde{\text{P}}$. Lemma 6 implies that Sj's equilibrium profit is

$$\pi_j \;=\; \frac{\widetilde{\Theta}}{\beta_j^S} \left[\frac{(\, 2 + \Omega_j \,)\, \Delta_{\widetilde{P}j}}{8 + \Omega_j} \right]^2 . \tag{61}$$

(60) and (61) imply that:

$$\pi_j \;>\; \pi_j^M \quad \Leftrightarrow \quad \widetilde{\Theta} \left[\Delta_{\widetilde{P}j} \right]^2 \;>\; \Theta \left[\Delta_{Pj} \right]^2 . \tag{62}$$

The last inequality in (62) holds because $\frac{\widetilde{\Theta}}{\Theta} > \phi_2 > 1$ and $\Delta_{\widetilde{P}j} > \Delta_{Pj}$ (due to $\frac{\widetilde{c}_j^P}{c_j^P} > 1$).

Case II. $\frac{\widetilde{\Theta}}{\Theta} \in (\, \phi_1, \phi_2 \,)$.

Proposition 2 implies that S1 competes against $\widetilde{\text{P}}$ whereas S2 faces no competition on P. Lemmas 3 and 6 imply that S1's equilibrium profit is $\pi_1 = \frac{\widetilde{\Theta}}{\beta_1^S} \left[\frac{(2+\Omega_1)\, \Delta_{\widetilde{P}1}}{8+\Omega_1} \right]^2$ and S2's equilibrium profit is $\pi_2 = \frac{\Theta \left[\widetilde{\Delta}_{P2} \right]^2}{16\, b_2^S}$. Therefore, (62) implies $\pi_1 > \pi_1^M$. (10) and (60) imply that:

$$\pi_2 \;>\; \pi_2^M \quad \Leftrightarrow \quad \left[\frac{\widetilde{\Delta}_{P2}}{\Delta_{P2}} \right]^2 \;>\; \left[\frac{4\sqrt{1 - \Omega_2}\, (\, 2 + \Omega_2 \,)}{8 + \Omega_2} \right]^2 . \tag{63}$$

Case III. $\frac{\widetilde{\Theta}}{\Theta} \in (\, 1, \phi_1 \,)$.

Proposition 2 implies that each seller sells on $\widetilde{\text{P}}$ and faces no competition. Lemma 3 implies that Sj's equilibrium profit is $\pi_j = \frac{\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}j}\right]^2}{16\,b_j^S}$. (60) implies that:

$$\pi_j \;>\; \pi_j^M \quad\Leftrightarrow\quad \frac{\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}j}\right]^2}{16\,b_j^S} \;>\; \frac{\Theta}{\beta_j^S}\left[\frac{(\,2+\Omega_j\,)\,\Delta_{Pj}}{8+\Omega_j}\right]^2. \tag{64}$$

(64) holds because

$$\frac{\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}j}\right]^2}{16\,b_j^S} \;>\; \frac{\Theta\left[\widetilde{\Delta}_{Pj}\right]^2}{16\,b_j^S} \;>\; \frac{\Theta}{\beta_j^S}\left[\frac{(\,2+\Omega_j\,)\,\Delta_{Pj}}{8+\Omega_j}\right]^2. \tag{65}$$

The first inequality in (65) reflects $\widetilde{\Theta} > \Theta$ and $\widetilde{\Delta}_{\widetilde{P}j} = \widetilde{\Delta}_{Pj}$ (due to $\frac{\partial\widetilde{\Delta}_{kj}}{\partial c_{kj}^P} = 0$). The second inequality in (65) reflects (63). Next suppose P faces a symmetric competing platform $\widetilde{\text{P}}$ (i.e., $\frac{\widetilde{\Theta}}{\Theta} = 1$). Proposition 2 implies that each seller faces no competition and is indifferent between selling on P and selling on $\widetilde{\text{P}}$ under PC. Lemma 3 implies that Sj's equilibrium profit is

$$\pi_j \;=\; \frac{1}{2}\frac{\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}j}\right]^2}{16\,b_j^S} + \frac{1}{2}\frac{\Theta\left[\widetilde{\Delta}_{Pj}\right]^2}{16\,b_j^S} \;>\; \frac{1}{2}\frac{\Theta\left[\widetilde{\Delta}_{Pj}\right]^2 + \Theta\left[\widetilde{\Delta}_{Pj}\right]^2}{16\,b_j^S} \;=\; \frac{\Theta\left[\widetilde{\Delta}_{Pj}\right]^2}{16\,b_j^S}. \tag{66}$$

The inequality in (66) reflects $\widetilde{\Theta} > \Theta$ and $\widetilde{\Delta}_{\widetilde{P}j} = \widetilde{\Delta}_{Pj}$ (due to $\frac{\partial\widetilde{\Delta}_{kj}}{\partial c_{kj}^P} = 0$). (60), (65), and (66) imply that $\pi_j > \pi_j^M$. Finally, suppose P faces a competing platform $\widetilde{\text{P}}$ that is a weaker platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} < 1$) under PC.

<u>Case I</u>. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, 1\right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\,1, \phi_1\,)$. Proposition 2 implies that each seller sells on P and faces no competition under PC. Lemma 3 implies that Sj's equilibrium profit is $\pi_j = \frac{\Theta\left[\widetilde{\Delta}_{Pj}\right]^2}{16\,b_j^S}$. (60) and (65) imply that $\pi_j > \pi_j^M$.

<u>Case II</u>. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1}\right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\,\phi_1, \phi_2\,)$. Proposition 2 implies that S1 competes against P whereas S2 sells on $\widetilde{\text{P}}$ and faces no competition under PC. Lemmas 3 and 6 imply that S1's equilibrium profit is $\pi_1 = \frac{\Theta}{\beta_1^S}\left[\frac{(\,2+\Omega_1\,)\,\Delta_{P1}}{8+\Omega_1}\right]^2$ and S2's equilibrium profit is $\pi_2 = \frac{\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{16\,b_2^S}$. (60) implies that $\pi_1 = \pi_1^M$. (64) implies that $\pi_2 > \pi_2^M$. ■

<u>Proof of Proposition 5</u>. (1) implies that if Sj sells on Pk and competes against Pk ($j, k \in$

$\{1,2\}$), then:

$$p^S_{kj} \;=\; \frac{q^P_{kj}}{\eta_j} - \frac{\theta_j\,\alpha_j}{\eta_j} + \frac{\beta^P_j\,p^P_{kj}}{\eta_j}. \tag{67}$$

(2) and (67) imply that:

$$p^P_{kj} \;=\; \frac{\eta_j\,q^S_{kj} + \beta^S_j\,q^P_{kj} - \alpha_j\left[\,\eta_j + \beta^S_j\,\theta_j\,\right]}{\left[\eta_j\right]^2 - \beta^S_j\,\beta^P_j}. \tag{68}$$

(67) and (68) imply that:

$$p^S_{kj} \;=\; \frac{\eta_j\,q^P_{kj} + \beta^P_j\,q^S_{kj} - \alpha_j\left[\,\theta_j\,\eta_j + \beta^P_j\,\right]}{\left[\eta_j\right]^2 - \beta^S_j\,\beta^P_j}. \tag{69}$$

Lemma 6 implies that if Sj competes against Pk, then:

$$q^{*S}_{kj} \;=\; \frac{Q^{*S}_{kj}}{\Theta_k} \;=\; \frac{[\,2+\Omega_j\,]\,\Delta_{kj}}{8+\Omega_j} \text{ and } q^{*P}_{kj} \;=\; \frac{Q^{*P}_{kj}}{\Theta_k} \;=\; \frac{\overline{\Delta}_{kj}}{2} + \frac{\eta_j\,[\,2+\Omega_j\,]\,\Delta_{kj}}{2\beta^S_j\,[\,8+\Omega_j\,]}. \tag{70}$$

(68), (69), and (70) imply that if S1 competes against Pk and S2 competes against Pi ($k, i \in \{1,2\}$), then consumer surplus is:

$$CS \;=\; \Theta_k\,\varsigma_{k1} + \Theta_i\,\varsigma_{i2}, \tag{71}$$

where for $j \in \{1,2\}$,

$$\varsigma_{kj} \;\equiv\; \frac{1}{\beta^S_j\,\beta^P_j\,[\,1-\Omega_j\,]}\left\{\frac{\eta_j\,[\,2+\Omega_j\,]\,\Delta_{kj}}{8+\Omega_j}\left[\frac{\overline{\Delta}_{kj}}{2} + \frac{\eta_j\,(\,2+\Omega_j\,)\,\Delta_{kj}}{2\beta^S_j\,(\,8+\Omega_j\,)}\right]\right.$$
$$\left. + \frac{\beta^S_j}{2}\left[\frac{\overline{\Delta}_{kj}}{2} + \frac{\eta_j\,(\,2+\Omega_j\,)\,\Delta_{kj}}{2\beta^S_j\,(\,8+\Omega_j\,)}\right]^2 + \frac{\beta^P_j}{2}\left[\frac{(\,2+\Omega_j\,)\,\Delta_{kj}}{8+\Omega_j}\right]^2\right\}. \tag{72}$$

It can be shown that:

$$\frac{\partial\varsigma_{kj}}{\partial c^P_{kj}} \;<\; 0. \tag{73}$$

(5) implies that if Sj sells on Pk ($j,k \in \{1,2\}$) and faces no competition, then:

$$q^S_{kj} \;=\; A_j - b^S_j\,p^S_{kj} \;\;\Leftrightarrow\;\; p^S_{kj} = \frac{A_j}{b^S_j} - \frac{q^S_{kj}}{b^S_j}. \tag{74}$$

Lemma 3 implies that if Sj sells on Pk ($k,i \in \{1,2\}$) and faces no competition, then:

$$q^{*S}_{kj} \;=\; \frac{Q^{*S}_j}{\Theta_k} \;=\; \frac{\widetilde{\Delta}_{kj}}{4}. \tag{75}$$

(74) and (75) imply that if S1 sells on Pk, S2 sells on Pi ($k, i \in \{1, 2\}$), and each seller faces no competition, then consumer surplus is:

$$CS \;=\; \frac{\Theta_k}{2\,\beta_1^S\,[\,1-\Omega_1\,]}\left[\frac{\widetilde{\Delta}_{k1}}{4}\right]^2 + \frac{\Theta_i}{2\,\beta_2^S\,[\,1-\Omega_2\,]}\left[\frac{\widetilde{\Delta}_{i2}}{4}\right]^2. \tag{76}$$

(68), (69), (70), (72), (74), and (75) imply that if S1 competes against Pk and S2 sells on Pi and faces no competition ($k, i \in \{1, 2\}$), then consumer surplus is:

$$CS \;=\; \Theta_k\,\varsigma_{k1} + \frac{\Theta_i}{2\,\beta_2^S\,[\,1-\Omega_2\,]}\left[\frac{\widetilde{\Delta}_{i2}}{4}\right]^2. \tag{77}$$

(12) and (72) imply that:

$$\varsigma_{kj} \;>\; \frac{1}{2\,\beta_j^S\,[\,1-\Omega_j\,]}\left[\frac{\widetilde{\Delta}_{kj}}{4}\right]^2. \tag{78}$$

Proposition 1 implies that each seller competes against P under MP. Therefore, (71) implies that:

$$CS^M \;=\; \Theta\,\varsigma_{P1} + \Theta\,\varsigma_{P2}. \tag{79}$$

First suppose P faces a competing platform $\widetilde{\text{P}}$ that is a stronger platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} > 1$) under PC, where $\widetilde{\Theta}$ denotes $\widetilde{\text{P}}$'s platform strength. (73) and $\frac{\widetilde{c}_j^P}{c_j^P} > 1$ imply that:

$$\frac{\varsigma_{Pj}}{\varsigma_{\widetilde{P}j}} \;>\; 1. \tag{80}$$

<u>Case I</u>. $\frac{\widetilde{\Theta}}{\Theta} > \max\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\}$.

Because $\frac{\widetilde{\Theta}}{\Theta} > \phi_{\widetilde{P}2}$, Proposition 2 implies that each seller competes against $\widetilde{\text{P}}$ under PC. Therefore, (71) implies that:

$$CS \;=\; \widetilde{\Theta}\,\varsigma_{\widetilde{P}1} + \widetilde{\Theta}\,\varsigma_{\widetilde{P}2}. \tag{81}$$

Because $\frac{\widetilde{\Theta}}{\Theta} > \frac{\varsigma_{Pj}}{\varsigma_{\widetilde{P}j}}$, $\widetilde{\Theta}\,\varsigma_{\widetilde{P}j} > \Theta\,\varsigma_{Pj}$. Therefore, (79) and (81) imply that $CS > CS^M$ in this case.

<u>Case II</u>. $\frac{\widetilde{\Theta}}{\Theta} \in \left(1, \min\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\}\right)$.

First suppose $\phi_{\widetilde{P}1} < \min\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}\right\}$. If $\frac{\widetilde{\Theta}}{\Theta} \in \left(\phi_{\widetilde{P}1}, \min\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\}\right)$, then $\frac{\widetilde{\Theta}}{\Theta} \in \left(\phi_{\widetilde{P}1}, \phi_{\widetilde{P}2}\right)$. Proposition 2 implies that S1 competes against $\widetilde{\text{P}}$ whereas S2 sells on P and faces no competition under PC. Therefore, (77) implies that:

$$CS \;=\; \widetilde{\Theta}\,\varsigma_{\widetilde{P}1} + \frac{\Theta}{2\,\beta_2^S\,[\,1-\Omega_2\,]}\left[\frac{\widetilde{\Delta}_{P2}}{4}\right]^2. \tag{82}$$

Because $\frac{\widetilde{\Theta}}{\Theta} < \frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}$, $\widetilde{\Theta}\,\varsigma_{\widetilde{P}1} < \Theta\,\varsigma_{P1}$. (78) implies that $\frac{1}{2\,\beta_2^S[1-\Omega_2]}\left[\frac{\Delta_{P2}}{4}\right]^2 < \varsigma_{P2}$. Therefore, (79) and (82) imply that $CS < CS^M$ in this case. If $\frac{\widetilde{\Theta}}{\Theta} \in \left(1, \phi_{\widetilde{P}1}\right)$, Proposition 2 implies that each seller sells on $\widetilde{\text{P}}$ and faces no competition under PC. Therefore, (76) implies that:

$$CS \;=\; \frac{\widetilde{\Theta}}{2\,\beta_1^S\,[\,1-\Omega_1\,]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}1}}{4}\right]^2 + \frac{\widetilde{\Theta}}{2\,\beta_2^S\,[\,1-\Omega_2\,]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}2}}{4}\right]^2. \tag{83}$$

(78) implies that $\frac{\widetilde{\Theta}}{2\,\beta_j^S[1-\Omega_j]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}j}}{4}\right]^2 < \widetilde{\Theta}\,\varsigma_{\widetilde{P}j}$. Because $\frac{\widetilde{\Theta}}{\Theta} < \frac{\varsigma_{Pj}}{\varsigma_{\widetilde{P}j}}$, $\widetilde{\Theta}\,\varsigma_{\widetilde{P}j} < \Theta\,\varsigma_{Pj}$. Therefore, $\frac{\widetilde{\Theta}}{2\,\beta_j^S[1-\Omega_j]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}j}}{4}\right]^2 < \Theta\,\varsigma_{Pj}$. Therefore, (79) and (83) imply that $CS < CS^M$ in this case.

Next suppose $\phi_{\widetilde{P}1} \geq \min\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}\right\}$. It can be shown that $\phi_{\widetilde{P}2} > \min\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}\right\}$. Therefore, $\min\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\} = \min\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}\right\}$. Because $\frac{\widetilde{\Theta}}{\Theta} \in \left(1, \min\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}\right\}\right)$, $\frac{\widetilde{\Theta}}{\Theta} \in \left(1, \phi_{\widetilde{P}1}\right)$. Proposition 2 implies that each seller sells on $\widetilde{\text{P}}$ and faces no competition under PC. Therefore, consumer surplus is given by (83). Therefore, (79) and (83) imply that $CS < CS^M$ in this case. Next suppose P faces a symmetric competing platform $\widetilde{\text{P}}$ (i.e., $\frac{\widetilde{\Theta}}{\Theta} = 1$) under PC. Proposition 2 implies that each seller is indifferent between selling on P and selling on $\widetilde{\text{P}}$ and each seller faces no competition under PC. Therefore, (76) implies that:

$$\begin{aligned} CS \;=\;& \frac{1}{2}\,\frac{\widetilde{\Theta}}{2\,\beta_1^S\,[\,1-\Omega_1\,]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}1}}{4}\right]^2 + \frac{1}{2}\,\frac{\Theta}{2\,\beta_1^S\,[\,1-\Omega_1\,]}\left[\frac{\widetilde{\Delta}_{P1}}{4}\right]^2 \\ &+ \frac{1}{2}\,\frac{\widetilde{\Theta}}{2\,\beta_2^S\,[\,1-\Omega_2\,]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}2}}{4}\right]^2 + \frac{1}{2}\,\frac{\Theta}{2\,\beta_2^S\,[\,1-\Omega_2\,]}\left[\frac{\widetilde{\Delta}_{P2}}{4}\right]^2. \end{aligned} \tag{84}$$

(78) implies that $\frac{\Theta}{2\,\beta_j^S[1-\Omega_j]}\left[\frac{\widetilde{\Delta}_{Pj}}{4}\right]^2 < \Theta\,\varsigma_{Pj}$ and $\frac{\widetilde{\Theta}}{2\,\beta_j^S[1-\Omega_j]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}j}}{4}\right]^2 < \widetilde{\Theta}\,\varsigma_{\widetilde{P}j}$. Because $\frac{\widetilde{\Theta}}{\Theta} = 1$, (80) implies that $\frac{\widetilde{\Theta}}{\Theta} < \frac{\varsigma_{Pj}}{\varsigma_{\widetilde{P}j}}$, and thus, $\widetilde{\Theta}\,\varsigma_{\widetilde{P}j} < \Theta\,\varsigma_{Pj}$. Therefore, $\frac{\widetilde{\Theta}}{2\,\beta_j^S[1-\Omega_j]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}j}}{4}\right]^2 < \Theta\,\varsigma_{Pj}$. Therefore, for $j \in \{1, 2\}$

$$\frac{1}{2}\,\frac{\widetilde{\Theta}}{2\,\beta_j^S\,[\,1-\Omega_j\,]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}j}}{4}\right]^2 + \frac{1}{2}\,\frac{\Theta}{2\,\beta_j^S\,[\,1-\Omega_j\,]}\left[\frac{\widetilde{\Delta}_{Pj}}{4}\right]^2 \;<\; \frac{\Theta}{2}\,\varsigma_{Pj} + \frac{\Theta}{2}\,\varsigma_{Pj} \;=\; \Theta\,\varsigma_{Pj}. \tag{85}$$

Therefore, (79), (84), and (85) imply that $CS < CS^M$ in this case. Finally, suppose P faces a competing platform $\widetilde{\text{P}}$ that is a weaker platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} < 1$) under PC.

<u>Case I</u>. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_{P1}}, 1\right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\,1, \phi_1\,)$. Proposition 2 implies that each seller sells on P and faces no competition under PC. Therefore, (76) implies that:

$$CS \ = \ \frac{\Theta}{2\,\beta_1^S\,[\,1-\Omega_1\,]}\left[\frac{\widetilde{\Delta}_{P1}}{4}\right]^2+\frac{\Theta}{2\,\beta_2^S\,[\,1-\Omega_2\,]}\left[\frac{\widetilde{\Delta}_{P2}}{4}\right]^2. \tag{86}$$

(78) implies that $\frac{1}{2\,\beta_j^S[1-\Omega_j]}\left[\frac{\widetilde{\Delta}_{Pj}}{4}\right]^2 < \varsigma_{Pj}$ for $j \in \{1,2\}$. Therefore, (79) and (86) imply that $CS < CS^M$ in this case.

Case II. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_{P2}}, \frac{1}{\phi_{P1}}\right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\,\phi_1, \phi_2\,)$. Proposition 2 implies that S1 competes against P whereas S2 sells on $\widetilde{\text{P}}$ and faces no competition under PC. Therefore, (77) implies that:

$$CS \ = \ \Theta\,\varsigma_{P1}+\frac{\widetilde{\Theta}}{2\,\beta_2^S\,[\,1-\Omega_2\,]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}2}}{4}\right]^2. \tag{87}$$

(78) implies that $\frac{\widetilde{\Theta}}{2\,\beta_2^S[1-\Omega_2]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}2}}{4}\right]^2 < \widetilde{\Theta}\,\varsigma_{\widetilde{P}2}$. (80) and $\frac{\widetilde{\Theta}}{\Theta} < 1$ imply that $\frac{\widetilde{\Theta}}{\Theta} < \frac{\varsigma_{Pj}}{\varsigma_{\widetilde{P}j}}$, and thus, $\widetilde{\Theta}\,\varsigma_{\widetilde{P}j} < \Theta\,\varsigma_{Pj}$. Therefore, $\frac{\widetilde{\Theta}}{2\,\beta_2^S[1-\Omega_2]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}2}}{4}\right]^2 < \Theta\,\varsigma_{P2}$. Therefore, (79) and (87) imply that $CS < CS^M$ in this case. ■

Proof of Proposition 6. (69) and (70) imply that if Sj sells on Pk and competes against Pk ($j,k \in \{1,2\}$), then in equilibrium:

$$p_{kj}^{*S} \ = \ \frac{1}{\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]}\left[\alpha_j\left(\,\theta_j\,\eta_j+\beta_j^P\,\right)-\frac{\eta_j\,\overline{\Delta}_{kj}}{2}-\frac{\beta_j^P\,(\,2+\Omega_j\,)^2\,\Delta_{kj}}{2\,(\,8+\Omega_j\,)}\right]. \tag{88}$$

(74) and (75) imply that if Sj sells on Pk ($j,k \in \{1,2\}$) and faces no competition, then in equilibrium:

$$p_{kj}^{*S} \ = \ \frac{A_j}{b_j^S}-\frac{\widetilde{\Delta}_{kj}}{4\,b_j^S} \ = \ \frac{4\,A_j-\widetilde{\Delta}_{kj}}{4\,b_j^S} \ = \ \frac{4\,A_j-\widetilde{\Delta}_{kj}}{4\,\beta_j^S\,[\,1-\Omega_j\,]}. \tag{89}$$

The last equality in (89) reflects (10). Let p_j^{SM} denote Sj's equilibrium price under MP and p_j^S denote Sj's equilibrium price under PC. Proposition 1 implies that Sj ($j \in \{1,2\}$) competes against P under MP. Therefore, (88) implies that:

$$p_j^{SM} \ = \ \frac{1}{\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]}\left[\alpha_j\left(\,\theta_j\,\eta_j+\beta_j^P\,\right)-\frac{\eta_j\,\overline{\Delta}_{Pj}}{2}-\frac{\beta_j^P\,(\,2+\Omega_j\,)^2\,\Delta_{Pj}}{2\,(\,8+\Omega_j\,)}\right]. \tag{90}$$

First suppose P faces a competing platform $\widetilde{\text{P}}$ that is a stronger platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} > 1$) under PC.

Case I. $\frac{\widetilde{\Theta}}{\Theta} > \phi_2$.

Proposition 2 implies that Sj ($j \in \{1,2\}$) competes against $\widetilde{\text{P}}$ under PC. Therefore, (88) implies that:

$$p_j^S \;=\; \frac{1}{\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]}\left[\alpha_j\left(\theta_j\,\eta_j+\beta_j^P\right)-\frac{\eta_j\,\overline{\Delta}_{\widetilde{P}j}}{2}-\frac{\beta_j^P\,(\,2+\Omega_j\,)^2\,\Delta_{\widetilde{P}j}}{2\,(\,8+\Omega_j\,)}\right]. \tag{91}$$

It can be shown that:

$$\frac{\partial\left(\frac{\eta_j\,\overline{\Delta}_{kj}}{2}+\frac{\beta_j^P(\,2+\Omega_j\,)^2\,\Delta_{kj}}{2(\,8+\Omega_j\,)}\right)}{\partial c_{kj}^P} \;<\; 0. \tag{92}$$

(90) - (92) imply that $p_j^S > p_j^{SM}$ in this case because $\frac{\widetilde{c}_j^P}{c_j^P} > 1$.

<u>Case II</u>. $\frac{\widetilde{\Theta}}{\Theta} \in (\,\phi_1,\phi_2\,)$.

Proposition 2 implies that S1 competes against $\widetilde{\text{P}}$ whereas S2 sells on P and faces no competition under PC. Therefore, (88) implies that:

$$p_1^S \;=\; \frac{1}{\beta_1^S\,\beta_1^P\,[\,1-\Omega_1\,]}\left[\alpha_1\left(\theta_1\,\eta_1+\beta_1^P\right)-\frac{\eta_1\,\overline{\Delta}_{\widetilde{P}1}}{2}-\frac{\beta_1^P\,(\,2+\Omega_1\,)^2\,\Delta_{\widetilde{P}1}}{2\,(\,8+\Omega_1\,)}\right]. \tag{93}$$

(90), (92), and (93) imply that $p_1^S > p_1^{SM}$ in this case because $\frac{\widetilde{c}_j^P}{c_j^P} > 1$. (89) implies that:

$$p_2^S \;=\; \frac{4\,A_2-\widetilde{\Delta}_{P2}}{4\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{94}$$

(90) and (94) imply that:

$$p_2^S \;>\; p_2^{SM}. \tag{95}$$

<u>Case III</u>. $\frac{\widetilde{\Theta}}{\Theta} \in (\,1,\phi_1\,)$.

Proposition 2 implies that Sj ($j \in \{1,2\}$) sells on $\widetilde{\text{P}}$ and faces no competition under PC. Therefore, (89) implies that:

$$p_j^S \;=\; \frac{4\,A_j-\widetilde{\Delta}_{\widetilde{P}j}}{4\,\beta_j^S\,[\,1-\Omega_j\,]}. \tag{96}$$

(53) and (95) imply that:

$$\frac{4\,A_j-\widetilde{\Delta}_{\widetilde{P}j}}{4\,\beta_j^S\,[\,1-\Omega_j\,]} \;=\; \frac{4\,A_j-\widetilde{\Delta}_{Pj}}{4\,\beta_j^S\,[\,1-\Omega_j\,]} \;>\; \frac{1}{\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]}\left[\alpha_j\left(\theta_j\,\eta_j+\beta_j^P\right)-\frac{\eta_j\,\overline{\Delta}_{Pj}}{2}-\frac{\beta_j^P\,(\,2+\Omega_j\,)^2\,\Delta_{Pj}}{2\,(\,8+\Omega_j\,)}\right]. \tag{97}$$

(90), (96), and (97) imply that $p_j^S > p_j^{SM}$ in this case. Next suppose P faces a symmetric competing platform $\widetilde{\text{P}}$ (i.e., $\frac{\widetilde{\Theta}}{\Theta} = 1$) under PC. Proposition 2 implies that Sj ($j \in \{1,2\}$) is indifferent between selling on P and selling on $\widetilde{\text{P}}$ and Sj faces no competition under PC. (89) implies that if Sj sells on P, then:

$$p_j^S \;=\; \frac{4\,A_j - \widetilde{\Delta}_{Pj}}{4\,\beta_j^S\,[\,1-\Omega_j\,]}; \tag{98}$$

if Sj sells on $\widetilde{\text{P}}$, then:

$$p_j^S \;=\; \frac{4\,A_j - \widetilde{\Delta}_{\widetilde{P}j}}{4\,\beta_j^S\,[\,1-\Omega_j\,]}. \tag{99}$$

(90), (97), (98), and (99) imply that $p_j^S > p_j^{SM}$ in this case. Finally, suppose P faces a competing platform $\widetilde{\text{P}}$ that is a weaker platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} < 1$) under PC.

<u>Case I</u>. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, 1\right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\,1, \phi_1\,)$. Proposition 2 implies that Sj ($j \in \{1,2\}$) sells on P and faces no competition under PC. Therefore, (89) implies that p_j^S is given by (98). (90), (97), and (98) imply that $p_j^S > p_j^{SM}$ in this case.

<u>Case II</u>. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1}\right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\,\phi_1, \phi_2\,)$. Proposition 2 implies that S1 competes against P whereas S2 sells on $\widetilde{\text{P}}$ and faces no competition under PC. Therefore, (88) implies that:

$$p_1^S \;=\; \frac{1}{\beta_1^S\,\beta_1^P\,[\,1-\Omega_1\,]}\left[\alpha_1\left(\,\theta_1\,\eta_1 + \beta_1^P\,\right) - \frac{\eta_1\,\overline{\Delta}_{P1}}{2} - \frac{\beta_1^P\,(\,2+\Omega_1\,)^2\,\Delta_{P1}}{2\,(\,8+\Omega_1\,)}\right]. \tag{100}$$

(90) and (100) imply that $p_1^S = p_1^{SM}$ in this case. (89) implies that:

$$p_2^S \;=\; \frac{4\,A_2 - \widetilde{\Delta}_{\widetilde{P}2}}{4\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{101}$$

(90), (97), and (101) imply that $p_2^S > p_2^{SM}$ in this case. ■

<u>Proof of Proposition 7</u>. Let w_j denote the commission Sj faces under PC and w_j^M denote the commission Sj faces under MP ($j \in \{1,2\}$). Proposition 1 implies that Sj ($j \in \{1,2\}$) competes against P under MP. Lemma 5 implies that:

$$w_j^M \;=\; \frac{1}{2\,[\,1-\Omega_j\,]}\left\{\frac{\Omega_j\,\overline{\Delta}_{Pj}}{\eta_j} + \frac{\left[\,8+(\Omega_j)^2\,\right]\,\Delta_{Pj}}{\beta_j^S\,[\,8+\Omega_j\,]}\right\}. \tag{102}$$

First suppose P faces a competing platform $\widetilde{\text{P}}$ that is a stronger platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} > 1$) under PC.

Case I. $\frac{\widetilde{\Theta}}{\Theta} > \phi_2$.

Proposition 2 implies that Sj ($j \in \{1,2\}$) competes against $\widetilde{\text{P}}$ under PC. Lemma 5 implies that:

$$w_j \;=\; \frac{1}{2\,[\,1-\Omega_j\,]} \left\{ \frac{\Omega_j\,\overline{\Delta}_{\widetilde{P}j}}{\eta_j} + \frac{\left[\,8+(\Omega_j)^2\,\right]\,\Delta_{\widetilde{P}j}}{\beta_j^S\,[\,8+\Omega_j\,]} \right\}. \tag{103}$$

It can be shown that:

$$\frac{\partial \left(\frac{\Omega_j\,\overline{\Delta}_{kj}}{\eta_j} + \frac{\left[8+(\Omega_j)^2\,\right]\Delta_{kj}}{\beta_j^S[\,8+\Omega_j\,]} \right)}{\partial c_{kj}^P} \;<\; 0. \tag{104}$$

(102) - (104) imply that $w_j < w_j^M$ if $\frac{\widetilde{\Theta}}{\Theta} > \phi_2$ because $\frac{\widetilde{c}_j^P}{c_j^P} > 1$.

Case II. $\frac{\widetilde{\Theta}}{\Theta} \in (\,\phi_1, \phi_2\,)$.

Proposition 2 implies that S1 competes against $\widetilde{\text{P}}$ whereas S2 sells on P and faces no competition under PC. Lemmas 2 and 5 imply that:

$$w_1 \;=\; \frac{1}{2\,[\,1-\Omega_1\,]} \left\{ \frac{\Omega_1\,\overline{\Delta}_{\widetilde{P}1}}{\eta_1} + \frac{\left[\,8+(\Omega_1)^2\,\right]\,\Delta_{\widetilde{P}1}}{\beta_1^S\,[\,8+\Omega_1\,]} \right\} \;\text{ and }\; w_2 \;=\; \frac{\widetilde{\Delta}_{P2}}{2\,b_2^S} \;=\; \frac{\widetilde{\Delta}_{P2}}{2\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{105}$$

The last equality in (105) reflects (10). (102), (104), and (105) imply that $w_1 < w_1^M$ if $\frac{\widetilde{\Theta}}{\Theta} \in (\,\phi_1, \phi_2\,)$ because $\frac{\widetilde{c}_1^P}{c_1^P} > 1$. (102) and (105) imply that:

$$w_2 \;>\; w_2^M. \tag{106}$$

Case III. $\frac{\widetilde{\Theta}}{\Theta} \in (\,1, \phi_1\,)$.

Proposition 2 implies that Sj ($j \in \{1,2\}$) sells on $\widetilde{\text{P}}$ and faces no competition under PC. Lemma 2 and (10) imply that:

$$w_j \;=\; \frac{\widetilde{\Delta}_{\widetilde{P}j}}{2\,b_j^S} \;=\; \frac{\widetilde{\Delta}_{\widetilde{P}j}}{2\,\beta_j^S\,[\,1-\Omega_j\,]}. \tag{107}$$

(53) and (106) imply that

$$\frac{\widetilde{\Delta}_{\widetilde{P}j}}{2\,\beta_j^S\,[\,1-\Omega_j\,]} \;=\; \frac{\widetilde{\Delta}_{Pj}}{2\,\beta_j^S\,[\,1-\Omega_j\,]} \;>\; \frac{1}{2\,[\,1-\Omega_j\,]} \left\{ \frac{\Omega_j\,\overline{\Delta}_{Pj}}{\eta_j} + \frac{\left[\,8+(\Omega_j)^2\,\right]\,\Delta_{Pj}}{\beta_j^S\,[\,8+\Omega_j\,]} \right\}. \tag{108}$$

Therefore, (102), (107), and (108) imply that $w_j > w_j^M$ if $\frac{\widetilde{\Theta}}{\Theta} \in (\,1, \phi_1\,)$. Next suppose P faces a symmetric competing platform $\widetilde{\text{P}}$ (i.e., $\frac{\widetilde{\Theta}}{\Theta} = 1$) under PC. Proposition 2 implies that Sj

($j \in \{1,2\}$) is indifferent between selling on P and selling on $\widetilde{\text{P}}$ and Sj faces no competition under PC. If Sj sells on P, Lemma 2 and (10) imply that:

$$w_j \;=\; \frac{\widetilde{\Delta}_{Pj}}{2\, b_j^S} \;=\; \frac{\widetilde{\Delta}_{Pj}}{2\, \beta_j^S \,[\, 1 - \Omega_j \,]}. \tag{109}$$

(102), (106), and (109) imply that $w_j > w_j^M$ in this case. If Sj sells on $\widetilde{\text{P}}$, Lemma 2 and (10) imply that w_j is given by (107). (102), (107), and (108) imply that $w_j > w_j^M$ in this case. Finally, suppose P faces a competing platform $\widetilde{\text{P}}$ that is a weaker platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} < 1$) under PC.

Case I. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, 1 \right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\, 1, \phi_1 \,)$. Proposition 2 implies that Sj ($j \in \{1,2\}$) sells on P and faces no competition under PC. Lemma 2 and (10) imply that w_j is given by (109). (102), (106), and (109) imply that $w_j > w_j^M$ if $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, 1 \right)$.

Case II. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1} \right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\, \phi_1, \phi_2 \,)$. Proposition 2 implies that S1 competes against P whereas S2 sells on $\widetilde{\text{P}}$ and faces no competition under PC. Lemmas 2 and 5 imply that:

$$w_1 \;=\; \frac{1}{2\,[\, 1 - \Omega_1 \,]} \left\{ \frac{\Omega_1\, \overline{\Delta}_{P1}}{\eta_1} + \frac{\left[\, 8 + (\Omega_1)^2 \,\right]\, \Delta_{P1}}{\beta_1^S \,[\, 8 + \Omega_1 \,]} \right\} \;\text{ and }\; w_2 \;=\; \frac{\widetilde{\Delta}_{\widetilde{P}2}}{2\, b_2^S} \;=\; \frac{\widetilde{\Delta}_{\widetilde{P}2}}{2\, \beta_2^S \,[\, 1 - \Omega_2 \,]}. \tag{110}$$

The last equality in (110) reflects (10). (102) and (110) imply that $w_1 = w_1^M$. (107). (102), (108), and (110) imply that $w_2 > w_2^M$. ■

Proof of Proposition 8. It can be shown that if S1 competes against Pk and S2 competes against Pi ($k, i \in \{1,2\}$), then social welfare is:

$$SW \;=\; \Theta_k\, \kappa_{k1} + \Theta_i\, \kappa_{i2} - 2\, F, \tag{111}$$

where

$$\kappa_{kj} \;\equiv\; \left[\frac{\overline{\Delta}_{kj}}{2} + \frac{\eta_j\,(\, 2 + \Omega_j \,)\, \Delta_{kj}}{2\beta_j^S\,(\, 8 + \Omega_j \,)} \right] \frac{\eta_j\, \Delta_{kj} + \beta_j^S\, \overline{\Delta}_{kj}}{\beta_j^S\, \beta_j^P \,[\, 1 - \Omega_j \,]} + \frac{[\, 2 + \Omega_j \,]\, \Delta_{kj}}{8 + \Omega_j} \frac{\beta_j^P\, \Delta_{kj} + \eta_j\, \overline{\Delta}_{kj}}{\beta_j^S\, \beta_j^P \,[\, 1 - \Omega_j \,]} - \varsigma_{kj}, \tag{112}$$

and ς_{kj} is given by (72). It can be shown that:

$$\kappa_{kj} \;=\; -\; \frac{\left[\overline{\Delta}_{kj} \right]^2}{8\, \beta_j^P \,[\, 1 - \Omega_j \,]} - \frac{[\, 2 + \Omega_j \,]^2\, [\, 4 + \Omega_j \,]\, [\Delta_{kj}]^2}{8\, \beta_j^S \,[\, 1 - \Omega_j \,]\, [\, 8 + \Omega_j \,]^2} - \frac{\eta_j\, [\, 2 + \Omega_j \,]\, \Delta_{kj}\, \overline{\Delta}_{kj}}{4\, \beta_j^P \,[\, 1 - \Omega_j \,]\, \beta_j^S \,[\, 8 + \Omega_j \,]}, \;\text{ and} \tag{113}$$

$$\frac{\partial \kappa_{kj}}{\partial c^P_{kj}} \; < \; 0. \tag{114}$$

Because $\frac{\widetilde{c}^P_j}{c^P_j} > 1$, (114) implies that:

$$\frac{\kappa_{Pj}}{\kappa_{\widetilde{P}j}} \; > \; 1. \tag{115}$$

(12) and (17) imply that:

$$\kappa_{kj} \; > \; \frac{7\left[\widetilde{\Delta}_{kj}\right]^2}{32\,\beta^S_j\,[\,1-\Omega_j\,]}.$$

It can be shown that if S1 sells on Pk, S2 sells on Pi ($k, i \in \{1, 2\}$), and each seller faces no competition, then social welfare is:

$$SW \; = \; \frac{7\,\Theta_k\left[\widetilde{\Delta}_{k1}\right]^2}{32\,\beta^S_1\,[\,1-\Omega_1\,]} + \frac{7\,\Theta_i\left[\widetilde{\Delta}_{i2}\right]^2}{32\,\beta^S_2\,[\,1-\Omega_2\,]}. \tag{116}$$

It can be shown that if S1 competes against Pk and S2 sells on Pi and faces no competition ($k, i \in \{1, 2\}$), then social welfare is:

$$SW \; = \; \Theta_k\,\kappa_{k1} + \frac{7\,\Theta_i\left[\widetilde{\Delta}_{i2}\right]^2}{32\,\beta^S_2\,[\,1-\Omega_2\,]} - F. \tag{117}$$

It can be shown that:

$$F \; < \; \Theta\,\kappa_{P1} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P1}\right]^2}{64\,\beta^S_1\,[\,1-\Omega_1\,]}; \tag{118}$$

$$F \; < \; \Theta\,\kappa_{P2} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta^S_2\,[\,1-\Omega_2\,]}. \tag{119}$$

Proposition 1 implies that each seller competes against P under MP. Therefore, (111) implies that:

$$SW^M \; = \; \Theta\,\kappa_{P1} + \Theta\,\kappa_{P2} - 2\,F, \tag{120}$$

First suppose P faces a competing platform $\widetilde{\text{P}}$ that is a stronger platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} > 1$) under PC.

<u>Case I</u>. $\frac{\widetilde{\Theta}}{\Theta} > \max\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\}$.

Because $\frac{\widetilde{\Theta}}{\Theta} > \phi_{\widetilde{P}2}$, Proposition 2 implies that each seller competes against $\widetilde{\text{P}}$ under PC. Therefore, (111) implies that:

$$SW \; = \; \widetilde{\Theta}\,\kappa_{\widetilde{P}1} + \widetilde{\Theta}\,\kappa_{\widetilde{P}2} - 2\,F, \tag{121}$$

Because $\frac{\widetilde{\Theta}}{\Theta} > \max\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}\right\}$, $\frac{\widetilde{\Theta}}{\Theta} > \frac{\kappa_{Pj}}{\kappa_{\widetilde{P}j}}$ ($j \in \{1,2\}$), and thus, $\widetilde{\Theta}\,\kappa_{\widetilde{P}j} > \Theta\,\kappa_{Pj}$. Therefore, (120) and (121) imply that $SW > SW^M$ in this case.

<u>Case II</u>. $\frac{\widetilde{\Theta}}{\Theta} \in \left(1, \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\}\right)$.

First suppose $\phi_{\widetilde{P}1} < \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}\right\}$. If $\frac{\widetilde{\Theta}}{\Theta} \in \left(\phi_{\widetilde{P}1}, \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\}\right)$, then $\frac{\widetilde{\Theta}}{\Theta} \in \left(\phi_{\widetilde{P}1}, \phi_{\widetilde{P}2}\right)$. Proposition 2 implies that S1 competes against $\widetilde{\text{P}}$ whereas S2 sells on P and faces no competition under PC. Therefore, (117) implies that:

$$SW \;=\; \widetilde{\Theta}\,\kappa_{\widetilde{P}1} + \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]} - F. \tag{122}$$

Because $\frac{\widetilde{\Theta}}{\Theta} < \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}\right\}$, $\frac{\widetilde{\Theta}}{\Theta} < \frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}$, and thus,

$$\widetilde{\Theta}\,\kappa_{\widetilde{P}1} \;<\; \Theta\,\kappa_{P1}. \tag{123}$$

It can be shown that $\frac{7\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]} < \kappa_{P2}$. Therefore, (79) and (82) imply that in this case:

$$SW \;<\; SW^M \;\;\Leftrightarrow\;\; F \;<\; \Theta\,\kappa_{P1} - \widetilde{\Theta}\,\kappa_{\widetilde{P}1} + \Theta\,\kappa_{P2} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{124}$$

(124) holds because (119) and (123) imply that $\Theta\,\kappa_{P1} - \widetilde{\Theta}\,\kappa_{\widetilde{P}1} > 0$, and thus,

$$F \;<\; \Theta\,\kappa_{P2} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]} \;<\; \Theta\,\kappa_{P1} - \widetilde{\Theta}\,\kappa_{\widetilde{P}1} + \Theta\,\kappa_{P2} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}.$$

If $\frac{\widetilde{\Theta}}{\Theta} \in \left(1, \phi_{\widetilde{P}1}\right)$, Proposition 2 implies that each seller sells on $\widetilde{\text{P}}$ and faces no competition under PC. Therefore, (116) implies that:

$$SW \;=\; \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{125}$$

It can be shown that $\frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}j}\right]^2}{32\,\beta_j^S\,[\,1-\Omega_j\,]} < \widetilde{\Theta}\,\kappa_{\widetilde{P}j}$. Because $\frac{\widetilde{\Theta}}{\Theta} < \phi_{\widetilde{P}1} < \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}\right\}$, $\frac{\widetilde{\Theta}}{\Theta} < \frac{\kappa_{Pj}}{\kappa_{\widetilde{P}j}}$ ($j \in \{1,2\}$), and thus,

$$\widetilde{\Theta}\,\kappa_{\widetilde{P}j} \;<\; \Theta\,\kappa_{Pj}. \tag{126}$$

Therefore, $\frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}j}\right]^2}{32\,\beta_j^S\,[\,1-\Omega_j\,]} < \Theta\,\kappa_{Pj}$. Condition FS ensures that $F \;<\; \widetilde{\Theta}\,M_{\widetilde{P}2} - \frac{\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{8\,b_2^S} = \widetilde{\Theta}\,M_{\widetilde{P}2} -$

$\frac{\widetilde{\Theta}[\widetilde{\Delta}_{\widetilde{P}2}]^2}{8\,\beta_2^S[1-\Omega_2]}$ and $F \;<\; \widetilde{\Theta}\,M_{\widetilde{P}1} - \frac{\widetilde{\Theta}[\widetilde{\Delta}_{\widetilde{P}1}]^2}{8\,b_1^S} = \widetilde{\Theta}\,M_{\widetilde{P}1} - \frac{\widetilde{\Theta}[\widetilde{\Delta}_{\widetilde{P}1}]^2}{8\,\beta_1^S[1-\Omega_1]}$. It can be shown that

$$F \;<\; \Theta\,\kappa_{P1} - \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} \;\text{ and }\; F \;<\; \Theta\,\kappa_{P2} - \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{127}$$

Therefore, (120) and (125) imply that in this case:

$$SW \;<\; SW^M \;\;\Leftrightarrow\;\; 2\,F \;<\; \Theta\,\kappa_{P1} - \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \Theta\,\kappa_{P2} - \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{128}$$

(127) implies that (128) holds. Next suppose $\phi_{\widetilde{P}1} \geq \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}\right\}$. (58) implies that $\phi_{\widetilde{P}2} > \phi_{\widetilde{P}1}$ and therefore $\phi_{\widetilde{P}2} > \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}\right\}$. Therefore, $\min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\} = \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}\right\}$. Therefore, $\frac{\widetilde{\Theta}}{\Theta} < \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\} = \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}\right\} \leq \phi_{\widetilde{P}1}$. Because $\frac{\widetilde{\Theta}}{\Theta} > 1$ in this case, then $\frac{\widetilde{\Theta}}{\Theta} \in \left(1, \phi_{\widetilde{P}1}\right)$. Proposition 2 implies that each seller sells on $\widetilde{\text{P}}$ and faces no competition under PC. Therefore, consumer surplus is given by (125). (120), (125), and (128) imply that $SW < SW^M$ in this case.

Next suppose P faces a symmetric competing platform $\widetilde{\text{P}}$ (i.e., $\frac{\widetilde{\Theta}}{\Theta} = 1$) under PC. Proposition 2 implies that each seller is indifferent between selling on P and selling on $\widetilde{\text{P}}$ and each seller faces no competition under PC. Therefore, (116) implies that:

$$SW \;=\; \frac{7\,\Theta\left[\widetilde{\Delta}_{P1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{129}$$

Therefore, (120) and (129) imply that in this case:

$$SW \;<\; SW^M \;\;\Leftrightarrow\;\; 2\,F \;<\; \Theta\,\kappa_{P1} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \Theta\,\kappa_{P2} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{130}$$

(118) and (119) imply that (130) holds. Finally, suppose P faces a competing platform $\widetilde{\text{P}}$ that is a weaker platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} < 1$) under PC.

<u>Case I</u>. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_{P1}}, 1\right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\,1, \phi_{P1}\,)$. Proposition 2 implies that each seller sells on P and faces no competition under PC. Therefore, (116) implies that:

$$SW \;=\; \frac{7\,\Theta\left[\widetilde{\Delta}_{P1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{131}$$

It can be shown that $\frac{7\left[\widetilde{\Delta}_{Pj}\right]^2}{32\,\beta_j^S\,[\,1-\Omega_j\,]} < \kappa_{Pj}$ for $j \in \{1,2\}$. Therefore, (120) and (131) imply that:

$$SW \;<\; SW^M \;\;\Leftrightarrow\;\; 2\,F \;<\; \Theta\,\kappa_{P1} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \Theta\,\kappa_{P2} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{132}$$

(118) and (119) imply that (132) holds.

Case II. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_{P2}}, \frac{1}{\phi_{P1}}\right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\,\phi_{P1}, \phi_{P2}\,)$. Proposition 2 implies that S1 competes against P whereas S2 sells on $\widetilde{\text{P}}$ and faces no competition under PC. Therefore, (117) implies that:

$$SW \;=\; \Theta\,\kappa_{P1} + \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]} - F. \tag{133}$$

It can be shown that $\frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]} < \Theta\,\kappa_{Pj}$. Therefore, (120) and (133) imply that in this case:

$$SW \;<\; SW^M \;\;\Leftrightarrow\;\; F \;<\; \Theta\,\kappa_{P2} - \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{134}$$

Because $\frac{\widetilde{\Theta}}{\Theta} < 1$ in this case and $\widetilde{\Delta}_{\widetilde{P}2} = \widetilde{\Delta}_{P2}$ (since $\frac{\partial \widetilde{\Delta}_{kj}}{\partial c_{kj}^P} = 0$), then $\Theta\,\kappa_{P2} - \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]} > \Theta\,\kappa_{P2} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}$. Therefore, (119) implies that (134) must hold. ■

Proof of Proposition 9. The proof proceeds in three steps. First, I show that under MP, both sellers sell on P, P does not make a no-entry commitment, and P charges its profit-maximizing commissions in equilibrium. Second, I show that under PC, if platforms have similar platform strengths, then no platform has an incentive to solely employ the commission instrument. Therefore, each platform either solely commits not to enter or employs both instruments in equilibrium under PC. Finally, I compare consumer surplus under MP and under PC. Under MP, each seller competes against P, P does not make a no-entry commitment, and P charges its profit-maximizing commissions in equilibrium. This is the case because each seller secures zero profit if it does not sell on P. Therefore, each seller sells on P that competes against sellers and charges its profit-maximizing commissions in equilibrium.[54] (79) implies that consumer surplus under MP is:

$$CS^M \;=\; \Theta\,\varsigma_{P1} + \Theta\,\varsigma_{P2}, \tag{135}$$

[54] Proposition 1 has shown that when the no-entry instrument is available, P has no incentive to employ it under MP. In addition, as the monopolistic platform, P is indifferent between committing to the profit-maximizing commission *ex ante* and setting it *ex post*.

Now suppose P faces a competing platform $\widetilde{\mathrm{P}}$ under PC. Lemma 3 implies that if Sj sells on Pk and Pk solely employs the no-entry instrument, then Sj's profit is:

$$\pi_{kj}^{N} = \frac{\Theta_k \left[\widetilde{\Delta}_{kj} \right]^2}{16\, b_j^S}, \tag{136}$$

and Pk's profit from the commission it collects from Sj is

$$\Pi_{kj}^{N} = \frac{\Theta_k \left[\widetilde{\Delta}_{kj} \right]^2}{8\, b_j^S}. \tag{137}$$

Lemma 4 implies that if Sj sells on Pk and Pk solely employs the commission instrument, then Sj's profit is:

$$\pi_{kj}^{C}(w_{kj}) = \frac{\Theta_k \left[\widetilde{\Delta}_{kj} + \Delta_{kj} - 2\, b_j^S\, w_{kj} \right]^2}{\beta_j^S \left[4 - \Omega_j \right]^2}, \tag{138}$$

and Pk's profit, which is the sum of its commission revenue from Sj and its retail revenue, is:

$$\Pi_{kj}^{C}(w_{kj}) = \left[p_{kj}^{P}(w_{kj}) - c_{kj}^{P} \right] Q_{kj}^{P}(w_{kj}) - F + w_{kj}\, Q_{kj}^{S}(w_{kj}),$$

where $w_{kj}^{01} \geq 0$ is the commission to which Pk commits before sellers make their platform choices.[55] Lemma 1 implies that if Sj sells on Pk and Pk employs both instruments, then Sj's profit is:

$$\pi_{kj}^{B}(w_{kj}) = \frac{\Theta_k \left[\widetilde{\Delta}_{kj} - b_j^S\, w_{kj} \right]^2}{4\, b_j^S} = \frac{\Theta_k \left[\widetilde{\Delta}_{kj} - b_j^S\, w_{kj} \right]^2}{4\, \beta_j^S \left[1 - \Omega_j \right]}, \tag{139}$$

and Pk's profit from the commission it collects from Sj is:

$$\Pi_{kj}^{B}(w_{kj}) = w_{kj}\, \frac{\Theta_k \left[\widetilde{\Delta}_{kj} - b_j^S\, w_{kj} \right]}{2}, \tag{140}$$

where $w_{kj} \geq 0$ is the commission to which Pk commits before sellers make their platform choices. (12) implies that $\frac{\partial \widetilde{\Delta}_{kj}}{\partial c_{kj}^P} = 0$. Therefore, for $k, i, j \in \{1, 2\}$ and $k \neq i$:

$$\widetilde{\Delta}_{ij} = \widetilde{\Delta}_{kj} = \widetilde{\Delta}_j. \tag{141}$$

[55] The superscript "N" in expression (136) denotes the setting in which Pk solely employs the no-entry instrument. Similarly, the superscript "C" in expression (138) denotes the setting in which Pk solely employs the commission instrument, and the superscript "B" in expression (139) denotes the setting in which Pk employs both instruments.

Because P and $\widetilde{\mathrm{P}}$ have similar platform strengths, it can be shown that

$$\frac{\widetilde{\Theta}}{\Theta} \ < \ \frac{[\,4-\Omega_j\,]^2 \left[\widetilde{\Delta}_j\right]^2}{16\,[\,1-\Omega_j\,] \left[\widetilde{\Delta}_j + \Delta_{\widetilde{P}j}\right]^2} \text{ and } \frac{\Theta}{\widetilde{\Theta}} \ < \ \frac{[\,4-\Omega_j\,]^2 \left[\widetilde{\Delta}_j\right]^2}{16\,[\,1-\Omega_j\,] \left[\widetilde{\Delta}_j + \Delta_{Pj}\right]^2}. \tag{142}$$

(10), (136), (138), and (141) imply that for $k, i, j \in \{1, 2\}$ and $k \neq i$:

$$\pi^C_{kj}(0) \ < \ \pi^N_{ij} \ \Leftrightarrow \ \frac{\Theta_k \left[\widetilde{\Delta}_{kj} + \Delta_{kj}\right]^2}{\beta^S_j\,[\,4-\Omega_j\,]^2} \ < \ \frac{\Theta_i \left[\widetilde{\Delta}_{ij}\right]^2}{16\,\beta^S_j\,[\,1-\Omega_j\,]} \ \Leftrightarrow \ \frac{\Theta_k}{\Theta_i} \ < \ \frac{[\,4-\Omega_j\,]^2 \left[\widetilde{\Delta}_j\right]^2}{16\,[\,1-\Omega_j\,] \left[\widetilde{\Delta}_j + \Delta_{kj}\right]^2}. \tag{143}$$

(143), $w_{kj} \geq 0$, and $\pi^C_{kj}(0) \geq \pi^C_{kj}(w_{kj})$ imply that:

$$\pi^C_{kj}(w_{kj}) \ < \ \pi^N_{ij} \ \Leftrightarrow \ \frac{\Theta_k}{\Theta_i} \ < \ \frac{[\,4-\Omega_j\,]^2 \left[\widetilde{\Delta}_j\right]^2}{16\,[\,1-\Omega_j\,] \left[\widetilde{\Delta}_j + \Delta_{kj}\right]^2}. \tag{144}$$

(142) and (144) imply that:

$$\pi^C_{\widetilde{P}j}\left(w_{\widetilde{P}j}\right) \ < \ \pi^N_{Pj} \ \text{ and } \ \pi^C_{Pj}(w_{Pj}) \ < \ \pi^N_{\widetilde{P}j}. \tag{145}$$

The first inequality in (145) implies that if $\widetilde{\mathrm{P}}$ solely employs the commission instrument, then P can successfully attract both sellers by committing not to enter. The last inequality in (145) implies that if P solely employs the commission instrument, then $\widetilde{\mathrm{P}}$ can successfully attract both sellers by committing not to enter. Consequently, no platform has an incentive to solely employ the commission instrument. Therefore, each platform either solely commits not to enter or employs both instruments in equilibrium under PC. Suppose S1 sells on Pk and S2 sells on Pi ($k, i \in \{1, 2\}$) in equilibrium. Because both sellers face no competition from platforms, it can be shown that consumer surplus under PC in this case is:

$$CS^C \ = \ \frac{\Theta_k}{2\,b^S_1}\left[\frac{\widetilde{\Delta}_1 - b^S_1\,w_{k1}}{2}\right]^2 + \frac{\Theta_i}{2\,b^S_2}\left[\frac{\widetilde{\Delta}_2 - b^S_2\,w_{i2}}{2}\right]^2, \tag{146}$$

where $w_{k1} \geq 0$ and $w_{i2} \geq 0$. (146), $w_{k1} \geq 0$, and $w_{i2} \geq 0$ imply that:

$$CS^C \ \leq \ \frac{\Theta_k}{2\,b^S_1}\left[\frac{\widetilde{\Delta}_1}{2}\right]^2 + \frac{\Theta_i}{2\,b^S_2}\left[\frac{\widetilde{\Delta}_2}{2}\right]^2. \tag{147}$$

Tedious calculations reveal that:

$$\varsigma_{kj} \ > \ \frac{1}{2\,b^S_j}\left[\frac{\widetilde{\Delta}_j}{2}\right]^2 \ \Leftrightarrow \ \frac{\overline{\Delta}_{kj}}{\Delta_{kj}} > \frac{-\overline{b} + \sqrt{\left(\overline{b}\right)^2 - 4\,\overline{a}\,\overline{c}}}{2\,\overline{a}}, \text{ and} \tag{148}$$

Case I. Both sellers sell on P in equilibrium under PC.

(146) and (147) imply that:

$$CS^C = \frac{\Theta}{2\,b_1^S}\left[\frac{\widetilde{\Delta}_1 - b_1^S\,w_{P1}}{2}\right]^2 + \frac{\Theta}{2\,b_2^S}\left[\frac{\widetilde{\Delta}_2 - b_2^S\,w_{P2}}{2}\right]^2 \leq \frac{\Theta}{2\,b_1^S}\left[\frac{\widetilde{\Delta}_1}{2}\right]^2 + \frac{\Theta}{2\,b_2^S}\left[\frac{\widetilde{\Delta}_2}{2}\right]^2. \quad (149)$$

(135), (149), and (148) imply that:

$$CS^C < CS^M \text{ if } \frac{\overline{\Delta}_{Pj}}{\Delta_{Pj}} > \frac{-\overline{b} + \sqrt{\left(\overline{b}\right)^2 - 4\,\overline{a}\,\overline{c}}}{2\,\overline{a}}. \quad (150)$$

Case II. Both sellers sell on $\widetilde{\text{P}}$ in equilibrium under PC.

(146) and (147) imply that:

$$CS^C = \frac{\widetilde{\Theta}}{2\,b_1^S}\left[\frac{\widetilde{\Delta}_1 - b_1^S\,w_{\widetilde{P}1}}{2}\right]^2 + \frac{\widetilde{\Theta}}{2\,b_2^S}\left[\frac{\widetilde{\Delta}_2 - b_2^S\,w_{\widetilde{P}2}}{2}\right]^2 \leq \frac{\widetilde{\Theta}}{2\,b_1^S}\left[\frac{\widetilde{\Delta}_1}{2}\right]^2 + \frac{\widetilde{\Theta}}{2\,b_2^S}\left[\frac{\widetilde{\Delta}_2}{2}\right]^2. \quad (151)$$

(135) and (151) imply that:

$$CS^C < CS^M \text{ if } \frac{\widetilde{\Theta}}{\Theta} < \frac{\varsigma_{Pj}}{\frac{1}{2\,b_j^S}\left[\frac{\widetilde{\Delta}_j}{2}\right]^2}. \quad (152)$$

Case III. Sj sells on P and Sl sells on $\widetilde{\text{P}}$ in equilibrium under PC ($j, l \in \{1, 2\}$, $j \neq l$).

(146) and (147) imply that:

$$CS^C = \frac{\Theta}{2\,b_j^S}\left[\frac{\widetilde{\Delta}_j - b_j^S\,w_{Pj}}{2}\right]^2 + \frac{\widetilde{\Theta}}{2\,b_l^S}\left[\frac{\widetilde{\Delta}_l - b_l^S\,w_{\widetilde{P}l}}{2}\right]^2 \leq \frac{\Theta}{2\,b_j^S}\left[\frac{\widetilde{\Delta}_j}{2}\right]^2 + \frac{\widetilde{\Theta}}{2\,b_l^S}\left[\frac{\widetilde{\Delta}_l}{2}\right]^2. \quad (153)$$

(135) and (153) imply that:

$$CS^C < CS^M \text{ if } \frac{\overline{\Delta}_{Pj}}{\Delta_{Pj}} > \frac{-\overline{b} + \sqrt{\left(\overline{b}\right)^2 - 4\,\overline{a}\,\overline{c}}}{2\,\overline{a}} \text{ and } \frac{\widetilde{\Theta}}{\Theta} < \frac{\varsigma_{Pl}}{\frac{1}{2\,b_l^S}\left[\frac{\widetilde{\Delta}_l}{2}\right]^2}. \quad \blacksquare \quad (154)$$

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